

Introduction

The use of ionizing radiation, genetic modification or chemicals to induce sterility in male insects is an ideal pest control method because it is harmless on the environment and on human health [2]. The effects of irradiation have been studied on hundreds of arthropods to identify cases where the technique can be applied [1]. We focus on insects with a relatively long adult lifespan, short reproductive stages and multiple matings, such as beetles (cockshafers, sweet potato weevil). We pose a mathematical model for the rising costs of crop damage due to infestation and the cost of producing sterile males in an effort to contain the invasion. We seek to minimize this cost in an optimal control setting and solve the problem using simulated annealing to obtain the optimal rate at which to release the sterile males.



Figure 1: Sweet potato weevil damage.

Model Formulation

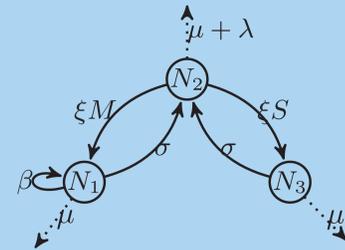
Assume a relationship between the change in cost, and the sum of production rate costs and feeding rate costs

$$\frac{dC(t)}{dt} = \underbrace{PX(t)}_{\text{Change in cost}} + \underbrace{FN(t)}_{\text{Losses to feeding rate}}$$

The previous equation governs the rate at which the cost is changing over time. Thus, we seek to minimize

$$\int_0^T PX(t) + FN(t) dt.$$

We consider the interactions between the stages of reproduction of the females to derive the state equations.



A large quantity of insects shall be released every d days. Thus, we discretize the portion of the integral that involves the control $X(t)$ into $\mathbf{X} = \{X_n\}_{n=0}^N$. Now we must seek

$$\min \sum_{n=0}^k PX_n + \int_0^T FN(t) dt$$

Total cost producing sterile males Total losses to feeding

subject to

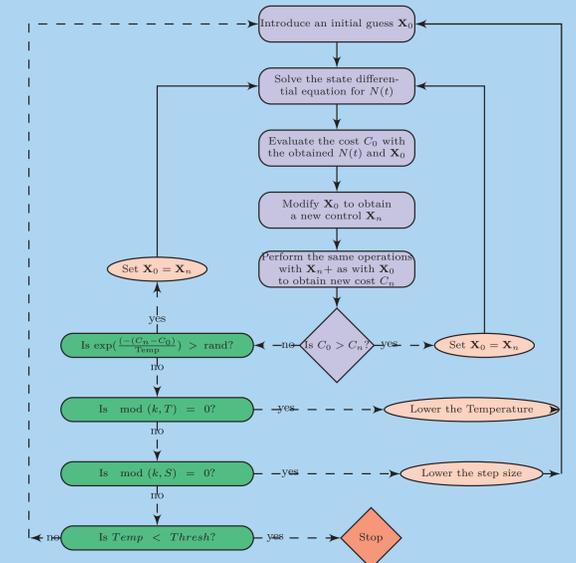
$$\frac{dN(t)}{dt} = \underbrace{\phi N(t)}_{\text{Natural growth rate}} \left(1 - \underbrace{\frac{\eta(1 + \theta S(t))}{1 + \gamma(N(t) + S(t))}}_{\text{Mating interruption}}\right)$$

$$\frac{dS(t)}{dt} = \underbrace{-\mu S(t)}_{\text{Natural death rate}} \text{ with } S(nd) = \underbrace{S(nd) + X_n}_{\text{Adding pulse of sterile males}}$$

On each interval $[nd, (n+1)d]$, over all permissible controls $\mathbf{X} = \{X_n\}_{n=0}^k$.

Simulated Annealing Algorithm

In order to solve this optimal control problem, we use the simulated annealing algorithm. The simulated annealing algorithm is a stochastic search method to find the minimum of a function.



Results

Typically, the optimal strategy to suppress the invasion was to produce sterile males at maximum capacity once the invasion begins. Once the invasive species population falls below a critical threshold, it can then taper off without any interference due to the Allee effect. In every case, the total cost was lower if the release dates were closer together.

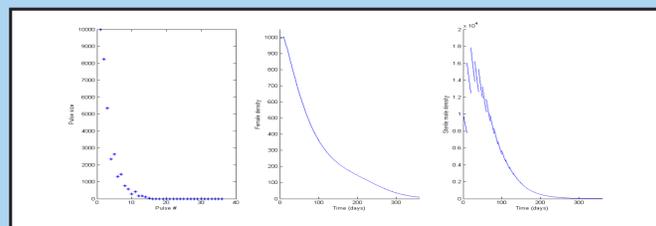


Figure 2: The optimal sterile insects release every ten days.

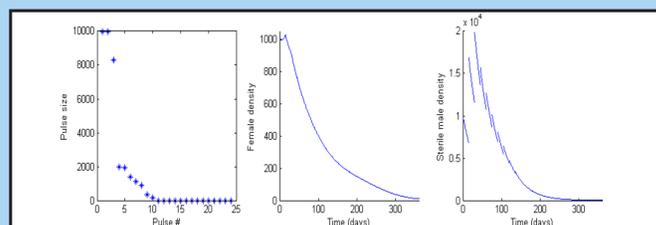


Figure 3: The optimal sterile insects release every 15 days.

We also experimented with different maximum production capacities. Observe that it is still important to produce at maximum capacity upon the beginning of the invasion. In every case we released sterile insects every ten days. A lower capacity requires a longer time to control the invasion.

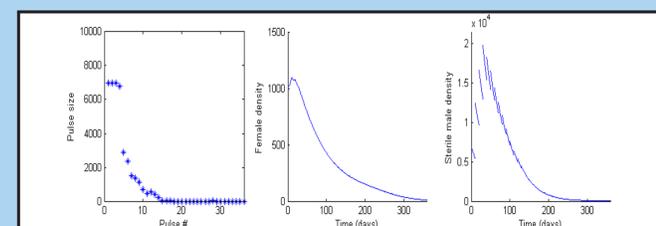


Figure 4: A ceiling of 7000 on production capacity.

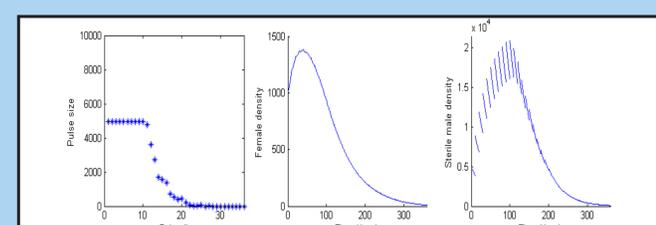


Figure 5: A maximum capacity of 5000.

Table of Parameters

Name	Description	value
β	Birth rate ($\frac{\text{insects}}{\text{day}}$)	0.3
μ	Death rate ($\frac{\text{insects}}{\text{day}}$)	.025
$1/\sigma$	Time spent pregnant	0.5
ξ	Per capita mating encounters	.0001
λ	Additional mortality term	0.0

Additional parameters:

$$\phi = \beta - \mu, \quad \eta = \frac{\sigma(\beta + \lambda)}{\phi(\sigma + \sigma)}$$

$$\gamma = \frac{\xi}{\sigma + \lambda}, \quad \theta = \frac{\beta\xi}{\sigma(\beta + \lambda)}$$

References

- [1] Bakri A., Mehta K., Lance D.R. (2005) Sterilizing insects with ionizing radiation. In "Sterile Insect Technique. Principles and Practice in Area-Wide Integrated Pest Management" (eds. V.A. Dyck, J. Hendrichs and A.S. Robinson), Springer, Netherlands, 233-258.
- [2] Klassen K., Curtis C.F. (2005) History of the sterile insect technique. In "Sterile Insect Technique. Principles and Practice in Area-Wide Integrated Pest Management" (eds. V.A. Dyck, J. Hendrichs and A.S. Robinson), Springer, Netherlands, 3-36.