CHAPTER 10 - REMINDER SHEET

The subject of Chapter 10 are first-order differential equations and models. First-order differential equations are very useful in *modeling* certain physical situations that involve "growth". Examples of these are population growth, radioactive decay, acceleration in a gravitational field, cooling, ...
Let $Q(t)$ be the amount of a measurable quantity (i.e. population, concentration, temperature, velocity,...) at a time t. Then

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \text{ the rate at which } Q \text{ varies over time.}$$

If we can describe the left hand side in terms of how this formula depends on t and Q itself then we get a first-order differential equation

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = F(t, Q).$$

For example, if we have a small object, Newton's Law of cooling for the temperature $Q(t)$ says that

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = k(Q - A),$$

where A is the ambient temperature and k is the product of the heat transfer coefficient and the heat transfer surface area. If the starting temperature of the object is above A, then we solve the equation for $Q(t) = A + Ce^{kt}$, where C is determined by the initial value of the equation. If the starting temperature of the object is below A, we arrive at the equation $Q(t) = A - Ce^{kt}$.
Finding the right formulation for F, i.e. the relation between the rate of change $\frac{\mathrm{d}Q}{\mathrm{d}t}$ and the time t and the quantity Q, is the main difficulty in setting up these problems.