CHAPTER 15 - REMINDER SHEET

The subject of Chapter 15 are solutions to second-order homogeneous linear differential equations with constant coefficients, i.e. differential equations of the form

$$a\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + b\frac{\mathrm{d}y}{\mathrm{d}x} + cy = 0,$$

where $a, b, c \in \mathbb{R}$ are constants such that $a \neq 0$. Then the *characteristic polynomial* for this equation is given as

$$ar^2 + br + c = 0.$$

There are three cases:

(1) There are two distinct real roots r_1 and r_2, i.e. $ar^2 + br + c = (r - r_1)(r - r_2)$. Then

$$\{e^{r_1 x}, e^{r_2 x}\}$$

is a fundamental set of solutions.

(2) There is only one real root r_1, i.e. $ar^2 + br + c = (r - r_1)^2$. Then

$$\{e^{r_1 x}, xe^{r_1 x}\}$$

is a fundamental set of solutions.

(3) There is a pair of conjugate complex roots $r_1 = \lambda + i\omega$ and $r_2 = \lambda - i\omega$. Then both

$$\{e^{(\lambda+i\omega)x}, e^{(\lambda-i\omega)x}\}$$

and

$$\{e^{\lambda x}\cos(\omega x), e^{\lambda x}\sin(\omega x)\}$$

are fundamental sets of solutions.

Example: Consider

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - y = 0.$$

Then $r^2 - 1 = 0$ splits as $(r-1)(r+1)$, hence we are in the first case. We get the fundamental set of solutions $\{e^x, e^{-x}\}$ we already got on the reminder sheet for Chapter 13.