CHAPTER 19 - REMINDER SHEET

The subject of Chapter 19 are nonhomogeneous linear equations of the form

$$\frac{\mathrm{d}^n y}{\mathrm{d}x^n} + a_1 \frac{\mathrm{d}^{n-1} y}{\mathrm{d}x^{n-1}} + \ldots + a_{n-1} \frac{\mathrm{d}y}{\mathrm{d}x} + a_n y = f(x),$$

where $a_1, \ldots, a_n$ are constant coefficients and $f(x) \neq 0$ is a continuous function. The corresponding homogeneous equation is determined by setting $f(x) = 0$.
For a homogeneous equation with two solutions y and $\tilde{y}$ the function $c_1 y + c_2 \tilde{y}$ is again a solution, as is easily checked. But for nonhomogeneous equations the sum of two solutions $c_1 y + c_2 \tilde{y}$ is not a solution, but rather a solution of the nonhomogeneous equation where the right hand side is $(c_1 + c_2) f(x)$. If we look at the case $c_1 = 1$ and $c_2 = -1$, then this yields a solution of the homogeneous equation. Thus the difference of two solutions of the nonhomogeneous equation is always a solution of the homogeneous equation.
If on the other hand y is a solution of the nonhomogeneous equation and $\tilde{y}$ is a solution of the homogeneous equation, then $y + \tilde{y}$ is again a solution of the nonhomogeneous equation.

Upshot: If we want to find a general solution to the nonhomogeneous equation, we should find a specific solution of the nonhomogeneous equation and a general solution of the homogeneous equation. Then their sum is a general solution of the non-homogeneous equation.