CHAPTER 20 - REMINDER SHEET

The subject of Chapter 20 is the method of undetermined coefficients. Take a nonhomogeneous differential equation

$$\frac{\mathrm{d}^n y}{\mathrm{d}x^n} + a_1 \frac{\mathrm{d}^{n-1} y}{\mathrm{d}x^{n-1}} + \ldots + a_{n-1} \frac{\mathrm{d}y}{\mathrm{d}x} + a_n y = f(x),$$

where $a_1, \ldots, a_n$ are constant coefficients and $f(x) \neq 0$ is a continuous function. In Chapter 19 we found out that a general solution to the equation is given by the sum of a general solution of the homogeneous equation and a particular solution of the nonhomogeneous equation.

The goal of the method of undetermined coefficients is finding such a particular solution of the nonhomogeneous equation.

Method of undetermined coefficients:

(1) Look at the function $f(x)$ and make a guess as to what the solution y could be. For example if f is an exponential function, then y should be of exponential form.

(2) We leave of all the coefficients of y undetermined and insert it into the differential equation.

(3) If we can find coefficients that satisfy the resulting equation, we have found a particular solution.

Example: Look at the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 3\frac{\mathrm{d}y}{\mathrm{d}x} - 4y = 3\exp 2x.$$

Then we guess that y is of the form $A \exp 2x$. Inserting this gives us

$$4A\exp 2x - 6A\exp 2x - 4A\exp 2x = 3\exp 2x,$$

hence $4A - 6A - 4A = 3$, i.e. $A = -\frac{1}{2}$. Thus we have the particular solution $-\frac{1}{2}\exp 2x$.

In general good guesses are

(1) $f(x)$ is an exponential function, then guess y is an exponential function.

(2) $f(x)$ is a function in cos or sin, then guess y is a sum of terms in sin and cos.

(3) $f(x)$ is a polynomial, then guess y is a polynomial of the same degree.

In case $f(x)$ is a product of functions of the above cases, we guess y to be a product of the above cases.