CHAPTER 23 - REMINDER SHEET

The subject of Chapter 23 is the *Laplace transform.* For a real-valued function $f : (0, \infty) \to \mathbb{R}$ the Laplace transform is defined as

$$F(s) = \mathscr{L}\{f(y)\}(s) = \int_0^\infty \exp(-st) f(t) dt.$$

Here s is a real parameter, even though in general it is defined to be a complex value. For us the Laplace transform is a useful tool to solve certain differential equations.

Example: The transform of $f(t) = t^n$ for a positive integer n is given as

$$\mathscr{L}\{t^n\}(s) = \int_0^\infty \exp(-st) t^n dt = \frac{n!}{s^{n+1}}.$$

From the shifting theorem

$$\mathscr{L}\{e^{at} f(t)\}(s) = \mathscr{L}\{f(t)\}(s-a), \qquad s > a$$

we can then deduce

$$\mathscr{L}\{e^{at} t^n\}(s) = \frac{n!}{(s-a)^{n+1}}$$

for $s > a$. Additional formulas include,

$$\mathscr{L}\{e^{at} \sin(kt)\}(s) = \frac{k}{(s-a)^2 + k^2}$$

and

$$\mathscr{L}\{e^{at} \cos(kt)\}(s) = \frac{(s-a)}{(s-a)^2 + k^2},$$

as well as

$$\mathscr{L}\{e^{at} \sinh(kt)\}(s) = \frac{k}{(s-a)^2 - k^2}$$

and

$$\mathscr{L}\{e^{at} \cosh(kt)\}(s) = \frac{(s-a)}{(s-a)^2 - k^2},$$

where always $s > a$ is assumed.