CHAPTER 27 - REMINDER SHEET

The subject of Chapter 27 are piecewise defined functions and their tranforms. In general a *piecewise defined function* is a function that is defined by several other functions which are defined on separate intervals of the domain.

Example: The *unit step function* $U : \mathbb{R} \to \mathbb{R}$ is defined as the step function that is given as

$$U(t) = \begin{cases} 0, & \text{if } t < 0 \\ 1, & \text{if } t \geq 0. \end{cases}$$

These functions as boundary conditions of differential equations can for example come up, when a force is acting, that can be switched on and off. We had already computed in Chapter 23, that the Laplace transform of the unit step function is given as $\frac{1}{s}\exp(-s)$.

Investigating further with shifted step functions $U(t - a)$, i.e.

$$U(t - a) = \begin{cases} 0, & \text{if } t < a \\ 1, & \text{if } t \geq a. \end{cases}$$

we deduce the *second shifting theorem*

$$\mathscr{L}\{f(t - a)U(t - a)\} = \exp(-as)F(s) \quad \Leftrightarrow \quad f(t - a)U(t - a) = \mathscr{L}^{-1}\{exp(-as)F(s)\}.$$

In words, muultiplying $F(s)$ by $\exp(-as)$ has the effect of shifting the funciton $f(t)$ by a units to the right in t-space and activating it a $t = a$.

Another special case of functions for which we compute the Laplace transform are periodic functions: A function $f : [0, \infty) \to \mathbb{R}$ is called *periodic of period* p if p is the least positive value such that $f(t + p) = f(t)$ for all $t \geq 0$. Then the Laplace transform of a periodic function f is equal to

$$\mathscr{L}\{f(t)\} = \frac{1}{1 - \exp(-sp)} \int_0^p \exp(-st)f(t)dt.$$