CHAPTER 6 - REMINDER SHEET

The subject of Chapter 6 are substitutions and how they help simplify differential equations.

1. Chapter 6.1 - General substitutions

Upshot: If a differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = F(x, y)$$

is not in one of the simple forms we have already seen in class (i.e. linear or separable equations), then we can try to use u-substitution to transform it into an easier form.

Example: If we want to start solving the equation

$$y\frac{\mathrm{d}y}{\mathrm{d}x} + x = \sqrt{x^2 + y^2}$$

then we see that if we make the substitution $u = x^2 + y^2$ we get

$$\frac{\mathrm{d}u}{\mathrm{d}x} = 2x + 2y\frac{\mathrm{d}y}{\mathrm{d}x}$$

and the substitution

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{u} - x}{y}$$

and thus

$$\frac{\mathrm{d}u}{\mathrm{d}x} = 2\sqrt{u}.$$

We know how to solve this separable equation, thus we have successfully simplified the original differential equation.

2. Chapters 6.2–6.4 - Useful special substitutions

Here are the three substitutions discussed in Chapters 6.2-6.4.

Name	Form of the DE	Substitution	Result
Linear Substitution	$\frac{\mathrm{d}y}{\mathrm{d}x} = F(Ax + By + C)$	$u = Ax + By + C$	$\frac{\mathrm{d}u}{\mathrm{d}x} = A + Bf(u)$
Homogeneous Equation	$\frac{\mathrm{d}y}{\mathrm{d}x} = F(\frac{y}{x})$	$u = \frac{y}{x}$	$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{f(u)-u}{x}$
Bernoulli Equations	$\frac{\mathrm{d}y}{\mathrm{d}x} + p(x)y = f(x)y^n$	$u = y^{1-n}$	$\frac{\mathrm{d}u}{\mathrm{d}x} + (1-n)p(x)u = (1-n)f(x)$

In particular in the case of linear substitutions and homogeneous substitutions we always end up with a separable differential equation and in the case of a Bernoulli equation we end up with a linear differential equation after substitution.