

CHAPTER 8 - REMINDER SHEET

The subject of Chapter 8 are slope fields. Sometimes it is not possible to solve the differential equation we are looking at, but we still have methods to analyze the solution qualitatively. One of these methods is investigating the slope field associated to the equation.

Let

$$\frac{dy}{dx} = f(y)$$

be an autonomous differential equation. Then the left hand side gives us the slope of every solution at any point in the XY-plane by evaluating f at that particular point. Using this, we can draw a plot of tangent lines to solution curves at any point in the plane called the *slope field*.

In the slope field in Figure 1 we can try to draw in graphs of solution curves using the slopes at every point.

Constant solutions and stability. A constant solution $y = y_0$ is called

- (1) *stable* if every other solution $y = y(x)$ having an initial value close to y_0 "stays close to y_0 ",
- (2) *asymptotically stable* if it is stable and the limit of any other solution "close to y_0 " is y_0 ,
- (3) *unstable* otherwise.

In Figure 1 we see one constant solution $y = 0$. Is this stable, asymptotically stable or unstable (see Exercise 8.8.9 d)?

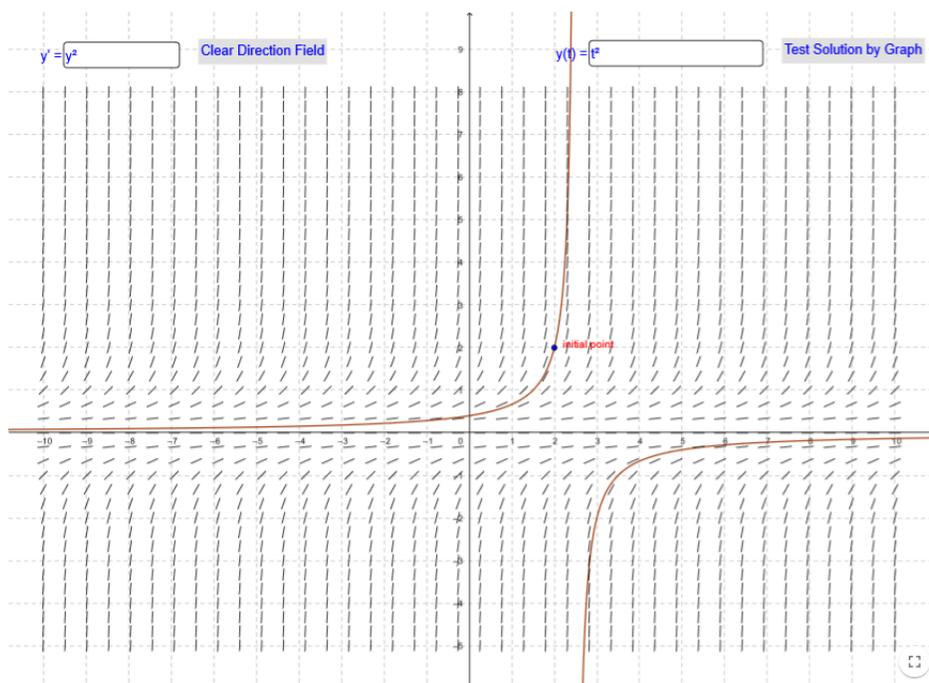


FIGURE 1. The slope field of the equation $\frac{dy}{dx} = y^2$