Math 1050 CBE 5 Review

UtahStateUniversity

Aggie**MathLearning**Center

CBE 5

- Covers lessons 23-26
  - Matrices and Systems of Equations
  - Nonlinear Systems of Equations
  - Partial Fraction Decomposition
  - Matrix Arithmetic

Matrices and Systems of Equations

Solving Linear Systems of Equations

Systems of Equations

- There are several different methods to solving systems of equations
- These methods include substitution and elimination

- Both methods follow the basic steps of:
  - Manipulate and solve for one variable
  - Plug that in to solve for the other variable

- Systems can be Independent (one solution), Dependent (many solutions), or inconsistent (no solution).

Problem 1

- Use any technique to determine the solution to the following system $-3x + 5y = 5$

$$5x - 5y = 1$$

- Solution: (3, 14/5)

Systems of Equations with Matrices

- Theorem:
  - If a system of equations has an augmented matrix that is row equivalent to the augmented matrix from another system of equations, then those systems have the same solution set
- Using the RREF of an augmented matrix makes solving linear systems of equations quick and easy.

Problem 2 – part 1

- The linear system of equations has the following RREF augmented matrix. Is the system independent, dependent, or inconsistent?

$$\begin{aligned} 5x + 4y + 3z &= 4 \\ 2x + y + 5z &= 6 \\ 4x + 3y + 2z &= 3 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

- Solution: Independent

Problem 2 – part 2

- Use the RREF to solve for x, y, and z.

$$5x + 4y + 3z = 4$$
$$2x + y + 5z = 6$$
$$4x + 3y + 2z = 3$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

- Solution: x = 1, y = -1, and z = 1

Problem 3

- The linear system of equations has the following RREF augmented matrix. Is the system independent, dependent, or inconsistent?

$$x + 2y + 3z = 11$$
$$2x + y + 9z = 5$$
$$3x + 5y + 10z = 33$$

$$\begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Solution: inconsistent

Problem 4 – part 1

- The linear system of equations has the following RREF augmented matrix. Is the system independent, dependent, or inconsistent?

$$
\begin{aligned}
2x + y - 3z &= 4 \\
6x + 11y - 49z &= 28 \\
x + 2y - 9z &= 5
\end{aligned}
\qquad
\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

- Solution: dependent

Problem 4 – part 2

- Use the RREF to solve for x, y, and z. Let z be the free variable, so z = z

$$2x + y - 3z = 4$$
$$6x + 11y - 49z = 28$$
$$x + 2y - 9z = 5$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- Solution: x = 1 - z, y = 2 – 5z, z = z

Nonlinear Systems of Equations

Setting up and Solving

Nonlinear Systems of Equations

- Again, there are multiple methods and ways to solve these systems, but the steps are similar to linear systems of equations and the basic steps are the same.

- Get equations in terms of only one variable
- Solve for the one variable
- Plug in solution to solve for the other variable

Problem 5

- Solve the following system of equations

$$-3y + 4x^2 - 2x = 3$$
$$4y + 3x = -4$$

- Solutions: x = -1/16, y = -61/64

  And x = 0, y = -1

Problem 6

- Solve the following system of equations

$$x^2 + y^2 = 25$$
$$x^2 + y^2 + 4y = 13$$

- Solution: x = 4, y = -3

  and x = -4, y = -3

Partial Fraction Decomposition

Set up, Use, and Formulas

Setting Up Partial Fraction Decomposition

- If the degree in the numerator is greater than the denominator use polynomial division
- Completely factor the denominator
- For each linear factor we need a fraction with a constant on top
  - Ex: $\dfrac{A}{bx+c}$
- For each quadratic factor we need a fraction with a linear equation on top
  - Ex: $\dfrac{Ax+B}{cx^2+dx+e}$
- For any squared or higher powered factors, we need a term for each power
  - Ex: $\dfrac{A}{(x+b)^3}+\dfrac{B}{(x+b)^2}+\dfrac{C}{(x+b)}$

Solving Partial Fraction Decomposition

- Multiply by least common multiple
- Group together like terms
- Set equal to original fraction
- Set up system of equations
- Solve for variables

Problem 7

- Write the following rational expression as a sum of partial fractions

$$\frac{4x + 16}{x^2 + 6x + 5}$$

- Solution: $\frac{1}{x+5} + \frac{3}{x+1}$

Problem 8

- Write the following rational expression as a sum of partial fractions

$$\frac{-4x^3 + 13x^2 + 49x - 62}{x^2 - 2x - 15}$$

- Solution: $-4x + 5 + \dfrac{1}{x-5} - \dfrac{2}{x+3}$

Matrix Arithmetic

Multiplication, Addition, and Subtraction

Addition and Subtraction

- Matrix addition and subtraction is element-wise
- In order to add matrices they must be the same size
- For example:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

Scalar Multiplication

- Scalar multiplication with matrices is also element-wise
- For example:

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

Matrix Multiplication

- In order to multiply matrices, they must have sizes MxN and NxK
  - (In other words the number of columns of the first matrix must equal the number of rows of the second)
- The element in the ith row and jth column of the product is found by multiplying the elements in the ith row of the first matrix with the corresponding elements in the jth column of the second matrix and adding those products together
- For Example:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

Problem 9

- Multiply the following matrices $$\begin{bmatrix} 4 & -2 \\ -9 & 2 \end{bmatrix} \times \begin{bmatrix} 10 & 4 \\ -9 & -10 \end{bmatrix}$$

- Solution: $$\begin{bmatrix} 58 & 36 \\ -108 & -56 \end{bmatrix}$$

Problem 10

- Perform the indicated operations on the following matrices

$$-\begin{bmatrix} -2 & 5 \\ 3 & 5 \end{bmatrix} + 2\begin{bmatrix} -3 & 0 \\ 5 & 3 \end{bmatrix}$$

- Solution: $\begin{bmatrix} -4 & -5 \\ 7 & 1 \end{bmatrix}$

Review Problem

Reviewing Solving Rational Equations

Problem 11

- Determine the real number solutions to the following rational equation $\frac{5x}{x-3} - \frac{28}{x^2-3x} = 4$

- Solution: x = -14 and 2

Other Resources

- Aggie Math Learning Center
  - Visit usu.edu/math/amlc for more videos, resources, tutoring times, and recitation leader office hours