Math 1050 Final Exam Review

Utah State University

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Final Exam

- Covers all lessons
  - This review will primarily focus on new material since CBE 4

Units 1-4 | 10 of the Problems

- Unit 1 Covers lessons 1-6
  - Intro to Complex Numbers
  - Solving Equations
  - Intro to Functions
  - Function Notation
  - Graphs of Functions
  - Graphs of Common Functions

- Unit 3 Covers lessons 11-16
  - Quadratic Functions
  - Polynomials and Their Graphs
  - Polynomial Division
  - Real Zeros of a Polynomial
  - Fundamental Theorem of Algebra
  - Rational Functions and Their Graphs

- Unit 2 Covers lessons 7-10
  - Circles
  - Transformations of Functions
  - Combining Functions
  - Inverse Functions

- CBE 4 Covers lessons 17-22
  - Solving rational inequalities
  - Exponential Functions
  - Logarithmic Functions and Properties of Logarithms
  - Solving Exponential and Logarithmic Equations
  - Exponential and Logarithmic Models

Unit 5 | 10 of the Problems

- Covers lessons 23-26
  - Matrices and Systems of Equations
  - Nonlinear Systems of Equations
  - Partial Fraction Decomposition
  - Matrix Arithmetic

Matrices and Systems of Equations

Solving Linear Systems of Equations

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Problem 1

- Use any technique to determine the solution to the following system $-3x + 5y = 5$

$$5x - 5y = 1$$

- Solution: (3, 14/5)

Systems of Equations with Matrices

- We can represent a system of equations with a matrix of the coefficients on each variable
- Using the Reduced Row-Echelon Form (RREF) of an augmented matrix highlights the solutions to the system – similar method to elimination.
  - If the bottom row of the RREF is all 0's, then the system is dependent
  - If there is a 1 and something else in the bottom right entries, then it is independent
  - When the bottom row is 0's except for the last entry, then it is inconsistent

Problem 2 – part 1

- The linear system of equations has the following RREF augmented matrix. Is the system independent, dependent, or inconsistent?

$$\begin{aligned} 5x + 4y + 3z &= 4 \\ 2x + y + 5z &= 6 \\ 4x + 3y + 2z &= 3 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

- Solution: Independent

Problem 2 – part 2

- Use the RREF to solve for x, y, and z.

$$5x + 4y + 3z = 4$$
$$2x + y + 5z = 6$$
$$4x + 3y + 2z = 3$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

- Solution: x = 1, y = -1, and z = 1

Problem 3

- The linear system of equations has the following RREF augmented matrix. Is the system independent, dependent, or inconsistent?

$$x + 2y + 3z = 11$$
$$2x + y + 9z = 5$$
$$3x + 5y + 10z = 33$$

$$\begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Solution: inconsistent

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Problem 4 – part 1

- The linear system of equations has the following RREF augmented matrix. Is the system independent, dependent, or inconsistent?

$$2x + y - 3z = 4$$
$$6x + 11y - 49z = 28$$
$$x + 2y - 9z = 5$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- Solution: dependent

Problem 4 – part 2

- Use the RREF to solve for x, y, and z. Let z be the free variable, so z = z

$$
\begin{aligned}
2x + y - 3z &= 4 \\
6x + 11y - 49z &= 28 \\
x + 2y - 9z &= 5
\end{aligned}
$$

$$
\begin{bmatrix}
1 & 0 & 1 & 1 \\
0 & 1 & -5 & 2 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

- Solution: x = 1 - z, y = 2 + 5z, z = z

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Nonlinear Systems of Equations

Setting up and Solving

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Problem 5

- Solve the following system of equations

$$x^2 + y^2 = 25$$
$$x^2 + y^2 + 4y = 13$$

- Solution: x = 4, y = -3

  and x = -4, y = -3

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Partial Fraction Decomposition

Set up, Use, and Formulas

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Setting Up Partial Fraction Decomposition

- Works when the degree of the numerator is less than the denominator (use division if not already)
- Completely factor the denominator
- For each linear factor we need a fraction with a constant on top
  - Ex: $\dfrac{A}{bx+c}$
- For each quadratic factor we need a fraction with a linear equation on top
  - Ex: $\dfrac{Ax+B}{cx^2+dx+e}$
- For any squared or higher powered factors, we need a term for each power
  - Ex: $\dfrac{A}{(x+b)^3}+\dfrac{B}{(x+b)^2}+\dfrac{C}{(x+b)}$

Solving Partial Fraction Decomposition

- Multiply each term by least common multiple of denominators
- Group together like terms in the numerator
- Set equal to original fraction
- Set up system of equations (variables are A, B, C, etc.)
- Solve for variables

Problem 6

- Write the following rational expression as a sum of partial fractions

$$\frac{4x+16}{x^2+6x+5}$$

- Solution: $\frac{1}{x+5} + \frac{3}{x+1}$

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Matrix Arithmetic

Multiplication, Addition, and Subtraction

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Matrix Arithmetic Operations

- Matrix Addition and Subtraction

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

- Scalar Multiplication

$$k \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

- Matrix Multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

Problem 7

- Multiply the following matrices $$\begin{bmatrix} 4 & -2 \\ -9 & 2 \end{bmatrix} \times \begin{bmatrix} 10 & 4 \\ -9 & -10 \end{bmatrix}$$

- Solution: $$\begin{bmatrix} 58 & 36 \\ -108 & -56 \end{bmatrix}$$

Problem 8

- Perform the indicated operations on the following matrices

$$-\begin{bmatrix} -2 & 5 \\ 3 & 5 \end{bmatrix} + 2\begin{bmatrix} -3 & 0 \\ 5 & 3 \end{bmatrix}$$

- Solution: $\begin{bmatrix} -4 & -5 \\ 7 & 1 \end{bmatrix}$

Functions

Notation, Domains, Ranges, and Inverses

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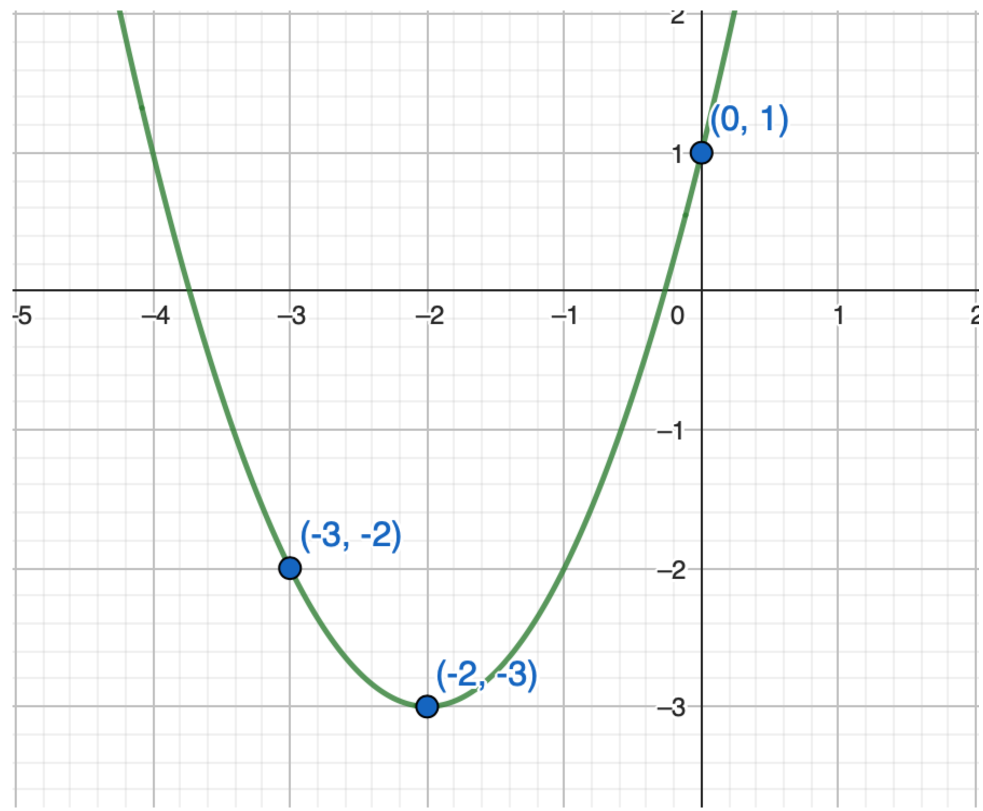
Problem 9

- Write the equation for the graph
  - Identify the parent graph
  - Identify transformations
  - Write known equation
  - Solve for unknowns
  - Finish equation
  - Solution: $y = (x+2)^2 - 3$



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Problem 10

$$f(x) = 3x^2 - 4x + 8$$

- Evaluate f(3) and f(x+h)

- Solutions: f(3) = 23 and f(x+h) = $3(x+h)^2 - 4(x+h) + 8$

Domain of a Function

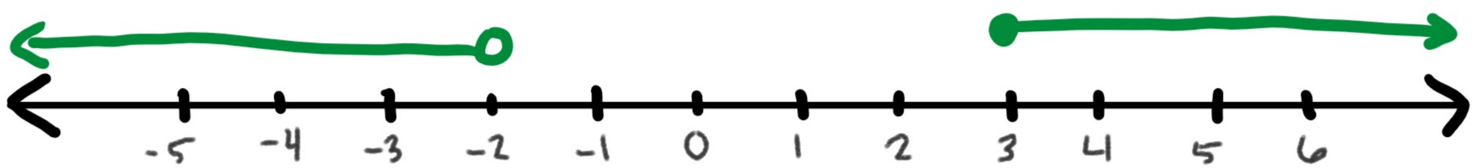
- All inputs of a function
  - Specifically, any real number input that has a real number output.

- When finding the domain of a function we usually find what values are not in the domain
- There are four things to look for when finding the domain:
  - Piecewise Functions – not always defined for all x
  - Fractions with x in the denominator – cannot equal 0
  - Square roots (Or any even powered root) with x – cannot be negative
  - Logs with x in them – cannot be ≤ 0

Interval Notation

- () means not included
- [] means included

- Write your intervals starting with the lowest number and ending with the greatest number, connecting intervals with U.



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Problem 11

- Use interval notation to write the domain of the following function:

$$h(x) = \frac{-3x-2}{\sqrt{2x+8}} - \frac{1}{6x-5} + \ln(x+3)$$

- Solution: (-3, 5/6) U (5/6, ∞)

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Combining Functions

- (f+g)(x) = f(x) + g(x)
  - For specific inputs add outputs
  - For general cases add functions
  - This applies for adding, subtracting, and multiplying

- (f/g)(x) = f(x)/g(x)
  - As long as g(x) does not equal 0
  - For specific inputs divide outputs
  - For general cases divide functions
  - ALWAYS check for g(x) = 0

Domains of Combined Functions

- Domain of (f + g)(x), (f – g)(x), (f x g)(x)
  - The domain is all of those for which f(x) and g(x) are both defined
  - Find the domains of f and g and keep whatever they have in common
- Domain of (f / g)(x)
  - Same as before, but x is not in the domain if g(x) = 0
  - Find the domains of f and g and keep whatever they have in common that does not make g(x) = 0

Composite Functions

- (f o g)(x) = f(g(x))
  - This means we will put our inputs into our function g, then we will put the outputs of that function into our function f and get our final output

- Domain of (f o g)(x)
  - Consists of all real numbers x such that x is in the domain of g and g(x) is in the domain of f
  - Find the domain of g, then decide which of those give outputs that are in the domain of f (and keep whatever is in both)

Problem 12 – Part 1

- Given $f(x) = \frac{1}{2x^2+1}$ and $g(x) = \sqrt{x - 3}$.

Find $(f + g)(4)$

and $(f \circ g)(6)$

Solutions: 34/33 and 1/7

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Problem 12 – Part 2

- Given $f(x) = \frac{1}{2x^2+1}$ and $g(x) = \sqrt{x - 3}$.

Find the domain of (f o g)(x)

Solution: $[3, \infty)$

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Inverse Functions

- Functions f and g are inverses if and only if f o g(x) = x and g o f(x) = x for all x in the domain of f and g.
  - A function must be one-to-one in order to be invertible
- We can find inverse functions by switching y and x in the equation and solving for y.

Problem 13 – Part 1

- Determine an expression for the inverse function of f

$$f(x) = 5\sqrt{2x+2} - 6$$

- Solution: $$\frac{(\frac{x+6}{5})^2 - 2}{2}$$

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Inverse Functions – Domains and Ranges

- The domain of the original function = the range of the inverse
- The range of the original function = the domain of the inverse
- We can find the domain of the inverse just like we would find the domain of any other function, however we then need to check if our original function imposes any restrictions

Problem 13 – Part 2

$$f(x) = 5\sqrt{2x+2} - 6$$

- Using interval notation write the domain and range for the inverse function of f

Solution:

Domain: $[-6, \infty)$ Range: $[-1, \infty)$

Polynomial Functions

Terminology, Zeros, and Polynomial Division

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Graphing Polynomials

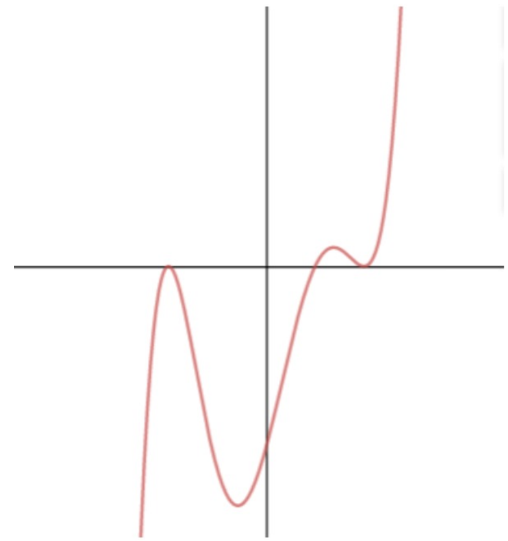
- In order to graph polynomials, we usually want to find the zeros and their multiplicities, and then look at end behavior

- End Behavior:
  - To determine the right end behavior, determine if the leading term of the polynomial is positive or negative for large positive values of x.

  - To determine the left end behavior, determine if the leading term of the polynomial is positive or negative for large negative values of x

Problem 14

- Choose the function from the following whose graph is shown, assume A,B,C,D are all positive real numbers



$$f(x) = -A(x+B)^2(x-C)(x-D)^2$$

$$h(x) = -A(x+B)(x-C)(x-D)$$

$$g(x) = A(x-B)^2(x+C)(x+D)^2$$

$$t(x) = A(x+B)^2(x-C)(x-D)^2$$

$$r(x) = -A(x-B)^3(x-C)(x-D)$$

- Solution: t(x)

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Graphing Rational Functions

- Holes:
  - Rational functions have holes for any input that is a zero of both the numerator and the denominator
  - Find holes by finding common factors of top and bottom to find the x coordinate and using the reduced form to find the y coordinate.

- Vertical Asymptotes:
  - Find vertical asymptotes by finding the zeros of the denominator of the reduced rational function

- Zeros:
  - Rational Functions have zeros when the numerator has a zero, but the denominator does not have a zero

Graphing Rational Functions

- End Behavior:
  - The end behavior is a horizontal asymptote if:
    - The degree in the numerator is less than the degree in the denominator
      - Then the horizontal asymptote is at y=0
    - The degree in the numerator is equal to the degree in the denominator
      - Then the horizontal asymptote is found by dividing the leading coefficients
  - The end behavior is an oblique (slant) asymptote if:
    - The degree in the numerator is exactly one greater than the degree in the denominator
      - Find the slant asymptote by dividing the numerator by the denominator and ignoring the remainder

Problem 15 – Part 1

- Consider the graph of the following function $f(x) = \frac{2x+5}{x^2-4}$.
- Does the graph have any vertical asymptotes? Where?

- Solution: Yes, x=2 and x=-2

Problem 15 – Part 2

- Consider the graph of the following function $f(x) = \frac{2x+5}{x^2-4}$.
- Does the graph have any horizontal asymptotes? Where?

- Solution: Yes, y=0

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Problem 15 – Part 3

- Consider the graph of the following function $f(x) = \frac{2x+5}{x^2-4}$.
- Where does the graph cross the x-axis?

- Solution: (-5/2, 0)

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Logarithms and Exponents

Properties, Solving Equations, and Models

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Problem 16

- Solve the following equation. Round your answer to four decimal points.

$$2\ln(x) + \ln(10) = 3$$

- Solution: 1.4172 or $\sqrt{\frac{e^3}{10}}$

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Other Resources

- Aggie Math Learning Center
  - Visit usu.edu/math/amlc for more videos, resources, tutoring times, and recitation leader office hours
  - Visit “Aggie Math” on YouTube for past CBE review recordings