# Math 1050 CBE 6 Review

**UtahState**University

Aggie**MathLearning**Center

# CBE 6

- Covers lessons 1-26
  - The CBE is a cumulative of all lessons from the semester

# Functions

Notation, Domains, Ranges, and Inverses

# Function Notation

- One way to think of functions is as a machine with inputs and outputs

- Functions are written as f(x) where x is a place holder that represents the input, and f represents the function itself and f(x) represents the output when the input is x.

- To evaluate a function at a value or to “plug in” a value we can replace “x” with the value we want to evaluate.

# Problem 1

$$f(x) = 3x^2 - 4x + 8$$

- Evaluate f(3) and f(x+h)

- Solutions: f(3) = 23 and f(x+h) = $3(x+h)^2 - 4(x+h) + 8$

# Domain of a Function

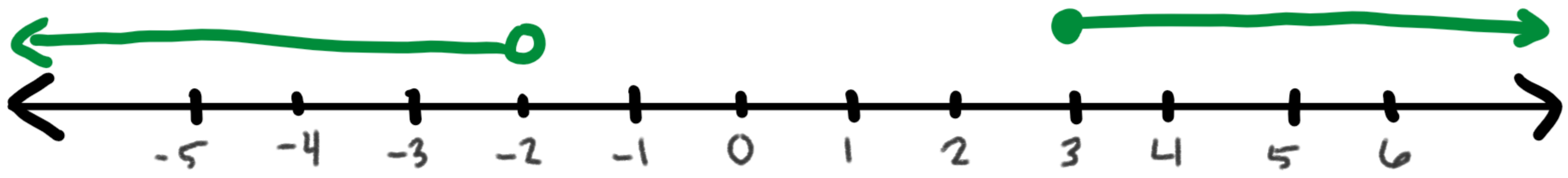
- All inputs of a function
  - Specifically, any real number input that has a real number output.

- When finding the domain of a function we usually find what values are not in the domain
- There are four things to look for when finding the domain:
  - Piecewise Functions
  - Fractions with x in the denominator
  - Square roots (Or any even powered root) with x in them
  - Logs with x in them

# Interval Notation

- () means not included
- [] means included

- Write your intervals starting with the lowest number and ending with the greatest number, connecting intervals with U.



# Problem 2

- Use interval notation to write the domain of the following function:

$$h(x) = \frac{-3x-2}{\sqrt{2x+8}} - \frac{1}{6x-5} + \ln(x+3)$$

- Solution: (-3, 5/6) U (5/6, ∞)

# Combining Functions

- $(f+g)(x) = f(x) + g(x)$
  - For specific inputs add outputs
  - For general cases add functions
  - This applies for adding, subtracting, and multiplying

- $(f/g)(x) = f(x)/g(x)$
  - As long as $g(x)$ does not equal 0
  - For specific inputs divide outputs
  - For general cases divide functions
  - ALWAYS check for $g(x) = 0$

# Domains of Combined Functions

- Domain of (f + g)(x), (f – g)(x), ( f x g)(x)
  - If and only if a real number x is in the domain of f and in the domain of g is x in the domain of (f+g), (f-g) and (f*g)
  - Find the domains of f and g and keep whatever they have in common
- Domain of (f / g)(x)
  - If and only if a real number x is in the domain of f and g, and g(x) is not equal to 0 is x in the domain of (f/g)
  - Find the domains of f and g and keep whatever they have in common that does not make g(x) = 0.

# Composite Functions

- (f o g)(x) = f(g(x))
  - This means we will put our inputs into our function g, then we will put the outputs of that function into our function f and get our final output

- Domain of (f o g)(x)
  - Consists of all real numbers x such that x is in the domain of g and g(x) is in the domain of f.
  - Find the domain of g, find the domain of the composite function and keep what they have in common

# Problem 3 – Part 1

- Given $f(x) = \frac{1}{2x^2+1}$ and $g(x) = \sqrt{x - 3}$.

Find $(f+g)(4)$

and $(f \circ g)(6)$

Solutions: 34/33 and 1/7

# Problem 3 – Part 2

- Given $f(x) = \frac{1}{2x^2+1}$ and $g(x) = \sqrt{x - 3}$.

Find the domain of (f o g)(x)

Solution: $[3, \infty)$

# Inverse Functions

- Functions f and g are inverses if and only if $f \circ g(x) = x$ and $g \circ f(x) = x$ for all x in the domain of f and g.
  - A function must be one-to-one in order to be invertible
- We can find inverse functions by switching y and x in the equation and solving for y.

# Problem 4 – Part 1

- Determine an expression for the inverse function of f

$$f(x) = 5\sqrt{2x+2} - 6$$

- Solution: $$\frac{(\frac{x+6}{5})^2 - 2}{2}$$

# Inverse Functions – Domains and Ranges

- The domain of the original function = the range of the inverse
- The range of the original function = the domain of the inverse
- We can find the domain of the inverse just like we would find the domain of any other function, however we then need to check if our original function imposes any restrictions

# Problem 4 – Part 2

$$f(x) = 5\sqrt{2x+2} - 6$$

- Using interval notation write the domain and range for the inverse function of f

Solution:

Domain: $[-6, \infty)$ Range: $[-1, \infty)$

# Graphing

Common Functions, Polynomials, and Rational Functions

# Common Graphs

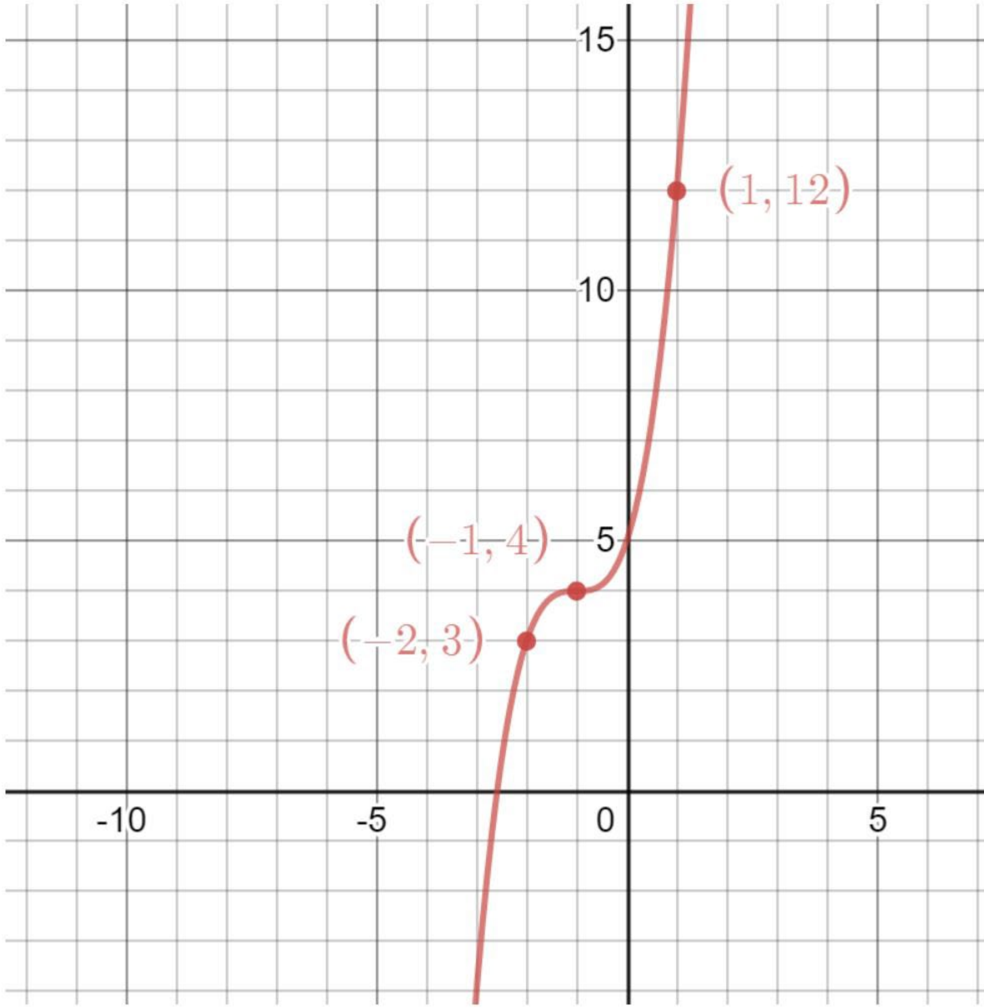
- Common Parent Graphs:
  - Linear $y = x$
  - Parabola $y = x^2$
  - Cubic $y = x^3$
  - Square Root $y = \sqrt{x}$
  - Absolute Value $y = |x|$
  - Circle $y^2 + x^2 = 1$

# Transformations

- Vertical Transformation $y = f(x) + k$
- Horizontal Transformation $y = f(x - h)$
- Vertical Reflection $y = -f(x)$
- Horizontal Reflection $y = f(-x)$
- Vertical Stretch $y = af(x)$
- Vertical Compression $y = \frac{1}{a}f(x)$

# Problem 5

- Write the equation for the graph shown




Solution: $f(x) = (x+1)^3 + 4$

# Polynomial Functions

Terminology, Zeros, and Polynomial Division

# Polynomials

- A polynomial is any function that *can* be written in the form:

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$$

- Leading Term: $a_n x^n$ Leading Coefficient: $a_n$
- Constant: $a_0$ Degree of the Polynomial: $n$

- If f(c) = 0 then c is said to be a zero of the function f.
- If (x-c) is a factor of f then c must be a zero of f.
- And if c is a zero of f then (x-c) must be a factor of f.
- If $(x-c)^n$ is a factor of f then c is a zero with multiplicity of n.

# Fundamental Theorem of Algebra

- If f(x) is a polynomial with real number coefficients and a leading term of $a_n x^n$ en f(x) can be written in the form $f(x) = a_n(x - c_1)(x - c_2)\ldots(x - c_n)$ where c1, c2,…,cn are complex numbers not necessarily unique.

- Polynomials of degree n have precisely n factors of the form (x-c)
- The sum of all multiplicities of all the zeros of a polynomial is equal to the degree

# Rational Zeros Theorem

- If the reduced rational number p/q is a zero of a polynomial, then p must be a factor of the constant term and q must be a factor of the leading coefficient

- In other words, any rational zeros must be a factor of the constant term divided by a factor of the leading coefficient.

# Graphing Polynomials

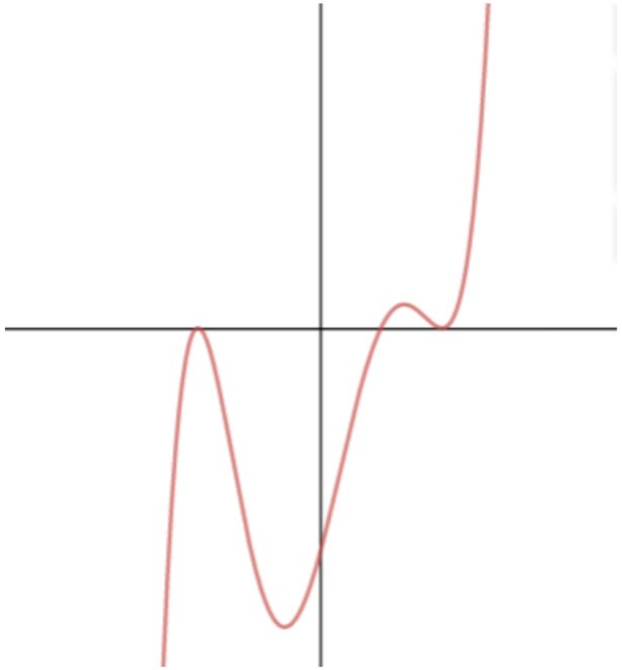
- In order to graph polynomials, we usually want to find the zeros and their multiplicities, and then look at end behavior

- End Behavior:
  - To determine the right end behavior, determine if the leading term of the polynomial is positive or negative for large positive values of x.

  - To determine the left end behavior, determine if the leading term of the polynomial is positive or negative for large negative values of x

# Problem 6

- Choose the function from the following whose graph is shown, assume A,B,C,D are all positive real numbers



$$f(x) = -A(x+B)^2(x-C)(x-D)^2$$

$$h(x) = -A(x+B)(x-C)(x-D)$$

$$g(x) = A(x-B)^2(x+C)(x+D)^2$$

$$t(x) = A(x+B)^2(x-C)(x-D)^2$$

$$r(x) = -A(x-B)^3(x-C)(x-D)$$

- Solution: t(x)

# Graphing Rational Functions

- When graphing rational functions, you want to look for and identify:
  - Holes
  - Vertical Asymptotes
  - End Behavior
  - Zeros

# Graphing Rational Functions

- Holes:
  - Rational functions have holes for any input that is a zero of both the numerator and the denominator
  - Find holes by finding common factors of top and bottom to find the x coordinate and using the reduced form to find the y coordinate.

- Vertical Asymptotes:
  - Find vertical asymptotes by finding the zeros of the denominator of the reduced rational function

- Zeros:
  - Rational Functions have zeros when the numerator has a zero, but the denominator does not have a zero

# Graphing Rational Functions

- End Behavior:
  - The end behavior is a horizontal asymptote if:
    - The degree in the numerator is less than the degree in the denominator
      - Then the horizontal asymptote is at y=0
    - The degree in the numerator is equal to the degree in the denominator
      - Then the horizontal asymptote is found by dividing the leading coefficients
  - The end behavior is an oblique (slant) asymptote if:
    - The degree in the numerator is exactly one greater than the degree in the denominator
      - Find the slant asymptote by dividing the numerator by the denominator and ignoring the remainder

# Problem 7 – Part 1

- Consider the graph of the following function $f(x) = \frac{2x+5}{x^2-4}$.
- Does the graph have any vertical asymptotes? Where?

- Solution: Yes, x=2 and x=-2

# Problem 7 – Part 2

- Consider the graph of the following function $f(x) = \frac{2x+5}{x^2-4}$.
- Does the graph have any horizontal asymptotes? Where?

- Solution: Yes, y=0

# Problem 7 – Part 3

- Consider the graph of the following function $f\left(x\right)=\frac{2x+5}{x^2-4}$.
- Where does the graph cross the x-axis?

- Solution: (-5/2, 0)

# Logarithms and Exponents

Properties, Solving Equations, and Models

# Properties of Exponents

- $a^m a^n = a^{m+n}$
- $(ab)^n = a^n b^n$
- $a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$ for all $a \neq 0$.

# Logarithms and properties of Logs

- The expression $\log_b a$ represents the number that you would raise b by to obtain a.
- $\log_e x$ is called a natural log and is written as ln(x)
- $\log_{10} x$ is called the common log and is written as log(x)

- Properties of Logs:

$$\log_a M + \log_a N = \log_a (MN)$$

$$\log_a M - \log_a N = \log_a \left(\frac{M}{N}\right)$$

$$r \log_a M = \log_a (M^r)$$

$$\log_b x = \frac{\log_c x}{\log_c b}$$

$$log_a a^x = x$$

$$a^{\log_a x} = x$$

# Solving Equations

- Solving exponential equations and logarithmic equations are really similar:
  - Use log/exponential properties to gather all logs/exponents into one
  - Set log/exponent = all else
  - Use appropriate exponentials or logs to “undo”
  - Solve for x

# Problem 8

- Solve the following equation. Round your answer to four decimal points.

$$2\ln(x) + \ln(10) = 3$$

- Solution: 1.4172

# Common Models

- Exponential Growth Model: $P(t) = P_0 e^{rt}$

- Exponential Decay Model: $A(t) = A_0 e^{-kt}$

- Using Models:
  - Identify what model to use and what variables are known.
  - Solve for unknown variables using given points
  - Use found variables to find wanted values

# Problem 9

- A biologist puts a population of 300 bacteria into a growth medium at 8 AM. She measures the sample again at 4 PM and now the population is 750 bacteria. How many bacteria will be there at 8 AM the next morning? Round your answer to the nearest whole number. (Assume the bacteria follows an exponential growth model)

- Solution: 4688 bacteria

# Partial Fraction Decomposition

Polynomial Division, Set Up, Solving

# Polynomial Division

- Long division, set up just like you were dividing two numbers

- Synthetic Division
  - Can only use when dividing by a zero. (x - c)
  - Set up with zero and coefficients
    - Don’t forget to write the zero coefficients

# Setting Up Partial Fraction Decomposition

- If the degree in the numerator is greater than the denominator use polynomial division
- Completely factor the denominator
- For each linear factor we need a fraction with a constant on top
  - Ex: $\frac{A}{bx+c}$
- For each quadratic factor we need a fraction with a linear equation on top
  - Ex: $\frac{Ax+B}{cx^2+dx+e}$
- For any squared or higher powered factors, we need a term for each power
  - Ex: $\frac{A}{(x+b)^3}+\frac{B}{(x+b)^2}+\frac{C}{(x+b)}$

# Solving Partial Fraction Decomposition

- Multiply by least common multiple
- Group together like terms
- Set equal to original fraction
- Set up system of equations
- Solve for variables

# Problem 10

- Write the following rational expression as a sum of partial fractions

$$\frac{3x^3 - 17x^2 + 28x - 14}{x^2 - 6x + 9}$$

- Solution: $3x + 1 + \frac{7}{x-3} - \frac{2}{(x-3)^2}$

# Other Resources

- Aggie Math Learning Center
  - Visit usu.edu/math/amlc for more videos, resources, tutoring times, and recitation leader office hours