Math 1050 CBE 2 Review

UtahStateUniversity

AggieMathLearningCenter

CBE 2

- Covers lessons 7-10
  - Circles
  - Transformations of Functions
  - Combining Functions
  - Inverse Functions

Transformations of Functions

shifts, stretches, compressions, and reflections

Transformations

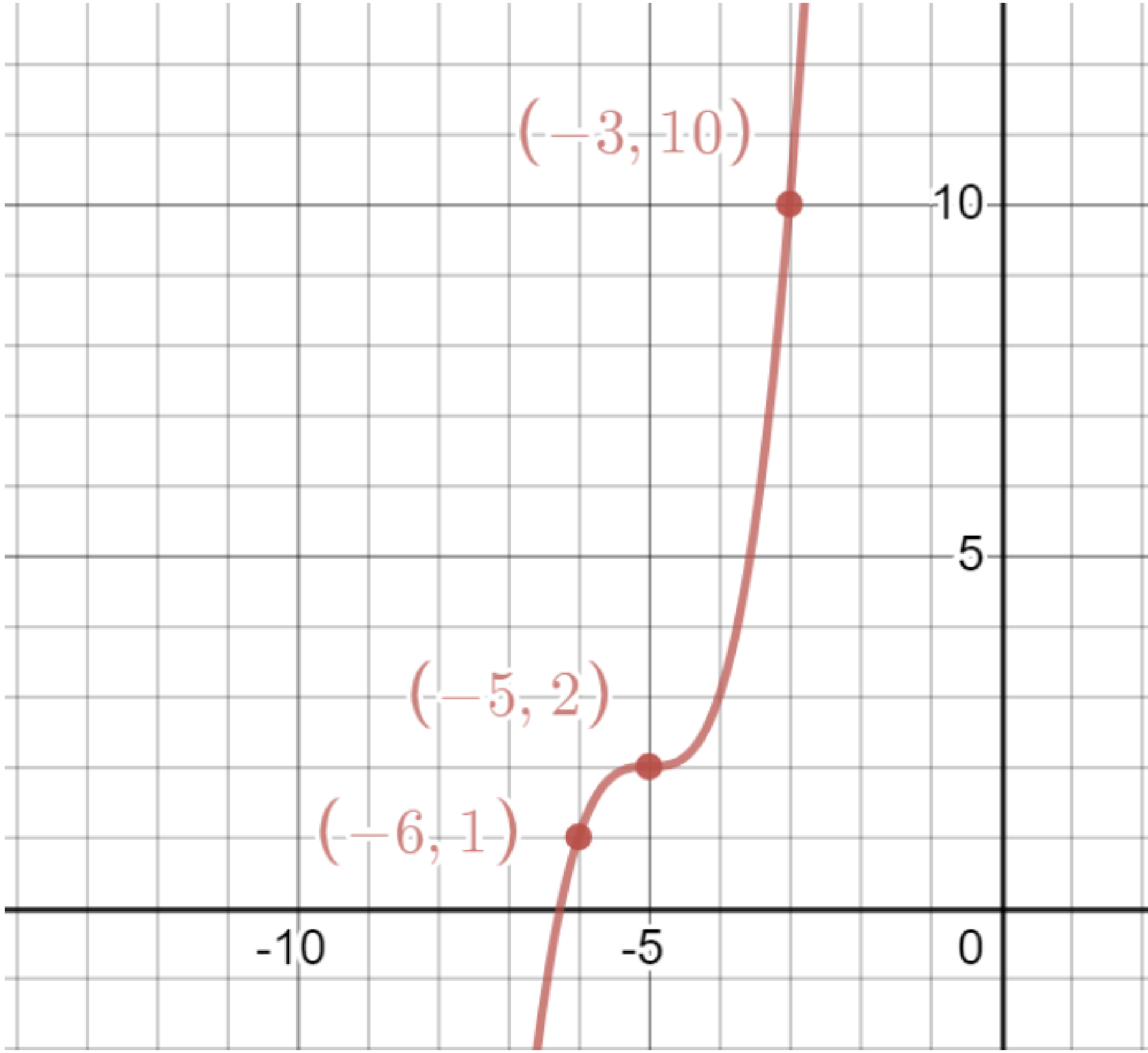
- Vertical Transformation $y = f(x) + k$
- Horizontal Transformation $y = f(x - h)$
- Vertical Reflection $y = -f(x)$
- Horizontal Reflection $y = f(-x)$
- Vertical Stretch $y = af(x)$
- Vertical Compression $y = \frac{1}{a} f(x)$

Common Parent Graphs

- Linear $y = x$
- Parabola $y = x^2$
- Cubic $y = x^3$
- Square Root $y = \sqrt{x}$
- Absolute Value $y = |x|$

Problem 1

- Write the equation for the graph
  - Identify the parent graph
  - Identify transformations
  - Write known equation
  - Solve for unknowns
  - Finish equation
  - Solution: $(x + 5)^3 + 2$



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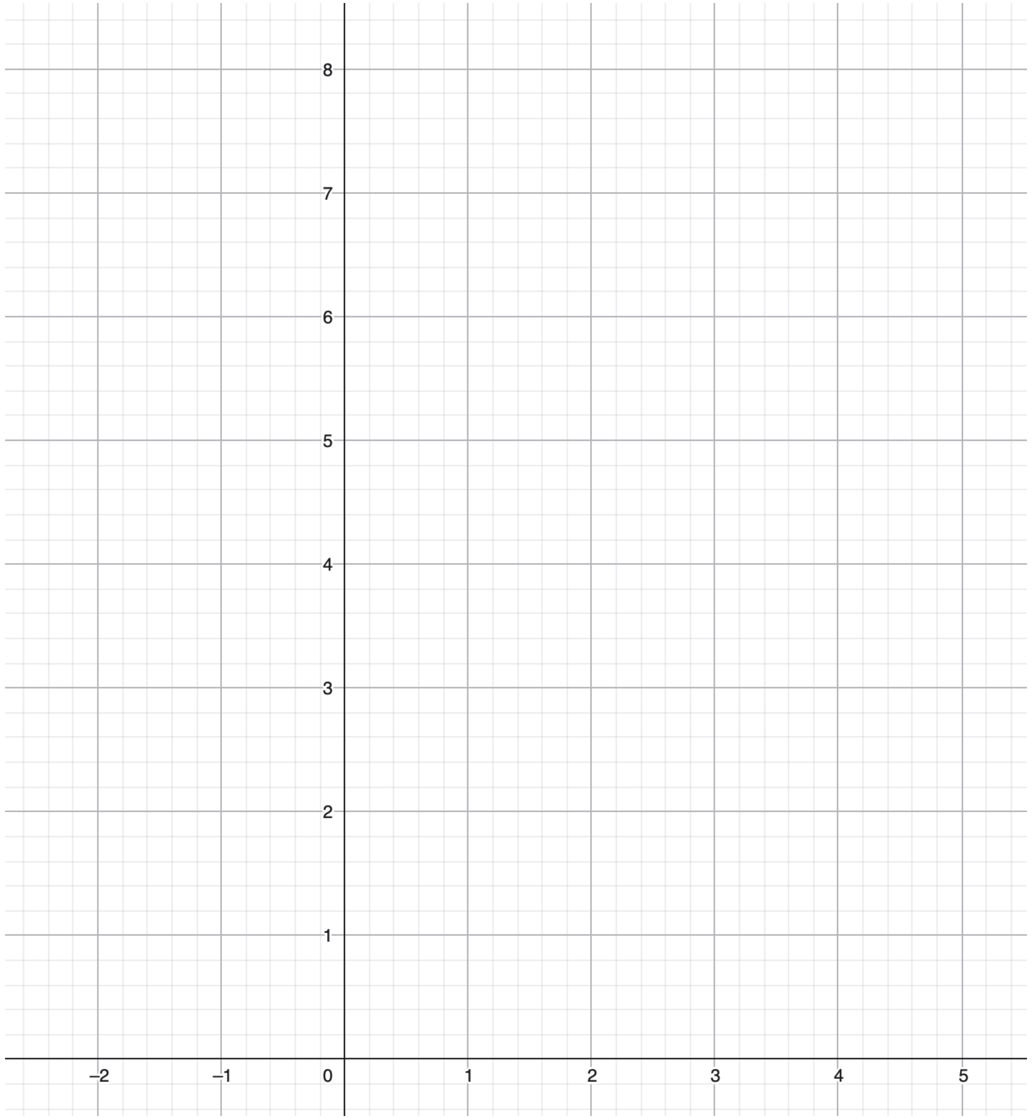
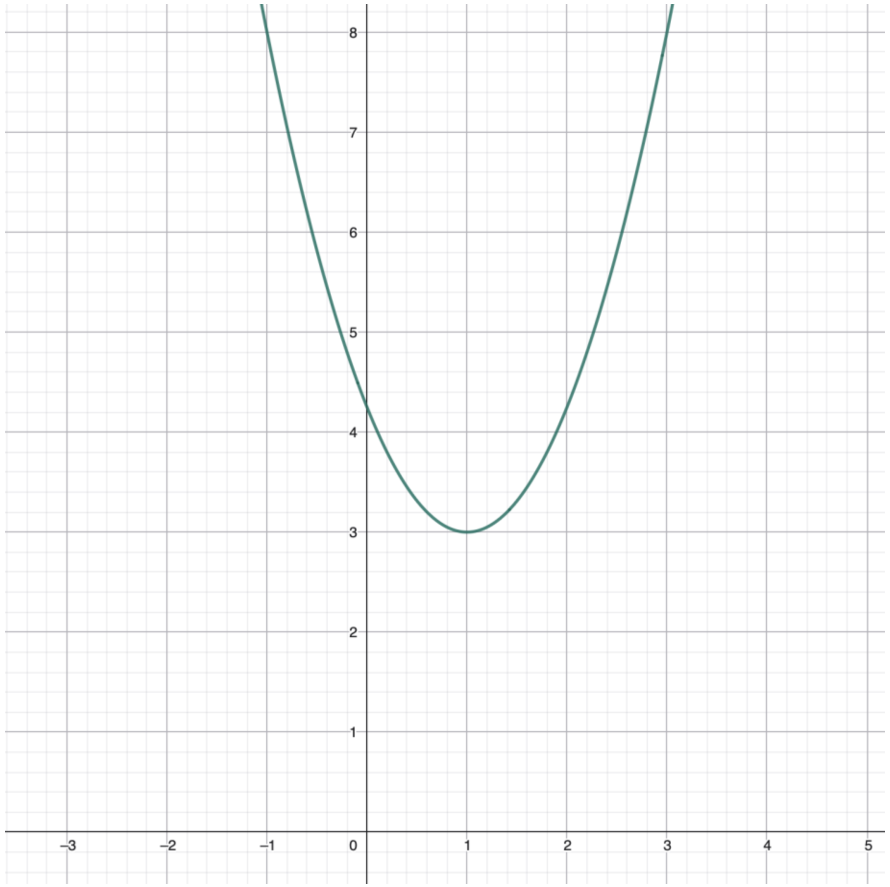
Problem 2

- Draw the graph of the function $f\left(x\right) = \frac{5}{4}\left(x-1\right)^2+3$





Problem 3

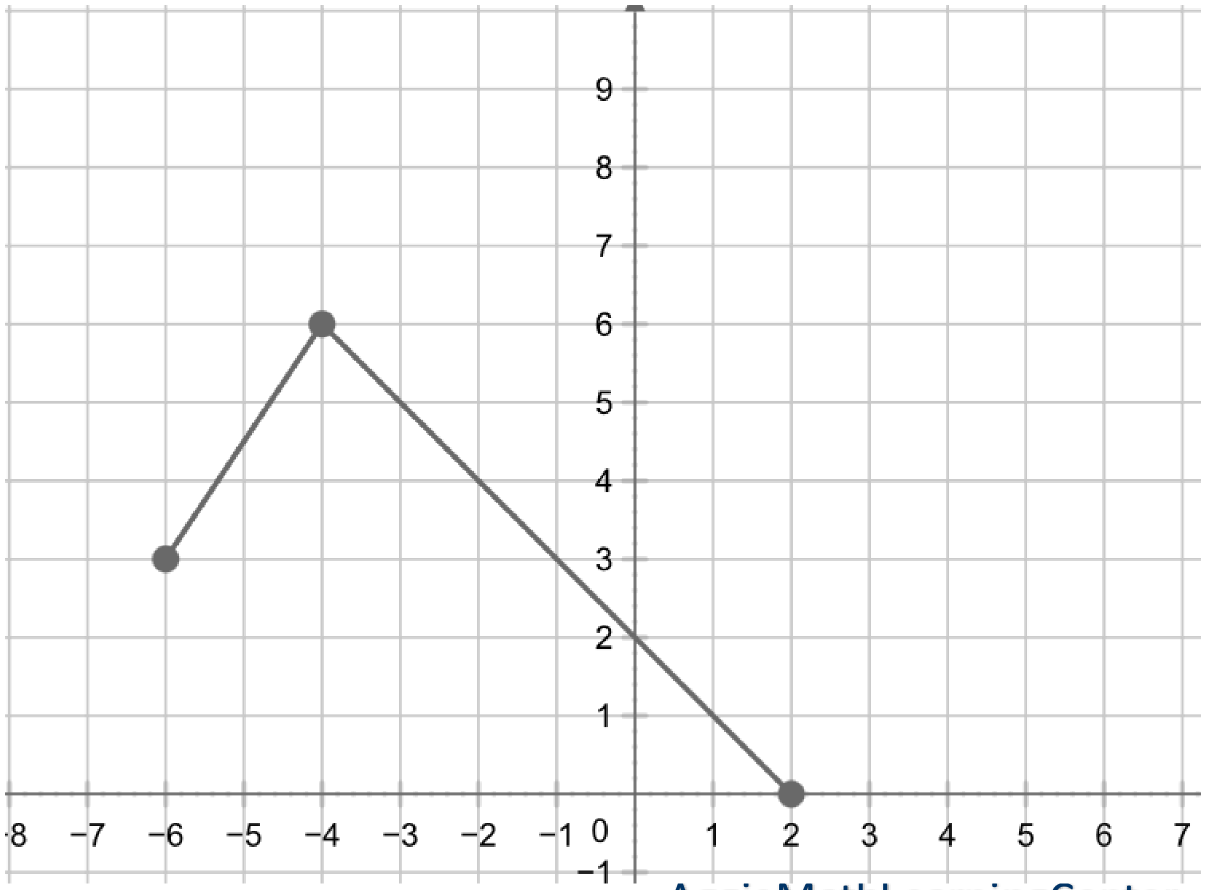
- The graph of the equation y = f(x) is shown.
- Draw the graph of the equation $y = \frac{1}{3}f(x-4)+2$

- Identify original points
- Identify transformations

- Apply Transformations

- Solution Points:

(-2,3), (0,4), (6,2)



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Circles

equation and fundamentals

Equation of a circle

$$(x-h)^2+(y-k)^2=r^2$$

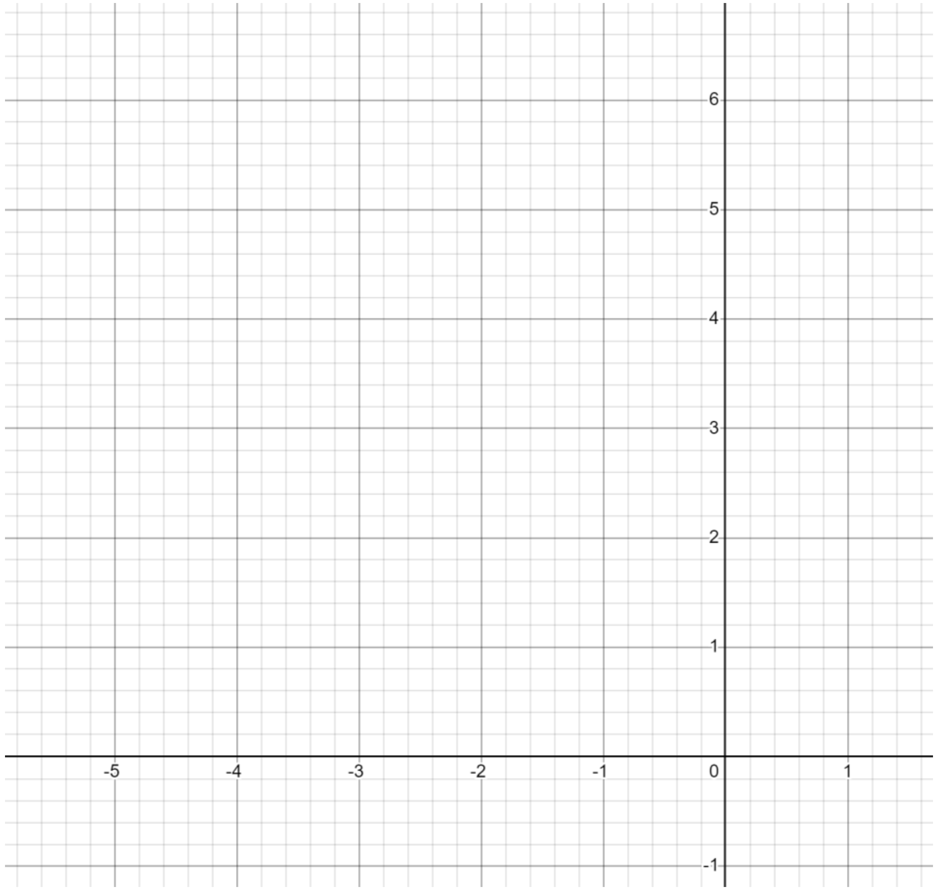
- The point (h, k) is the center of the circle
- r is the radius of the circle

Problem 4

- Graph the circle defined by the equation

$$(x+3)^2 + (y-3)^2 = 4$$

  - Identify center
  - Identify radius
  - Plot points
  - Connect points



  - Solution:
    Center: (-3,3)
    Radius: 2

Distance Formula

- The distance between two points (x_1, y_1) and (x_2, y_2) is given by the following formula
- Note that the distance is the same irrespective of the order (as squaring will always make it positive)

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Midpoint Formula

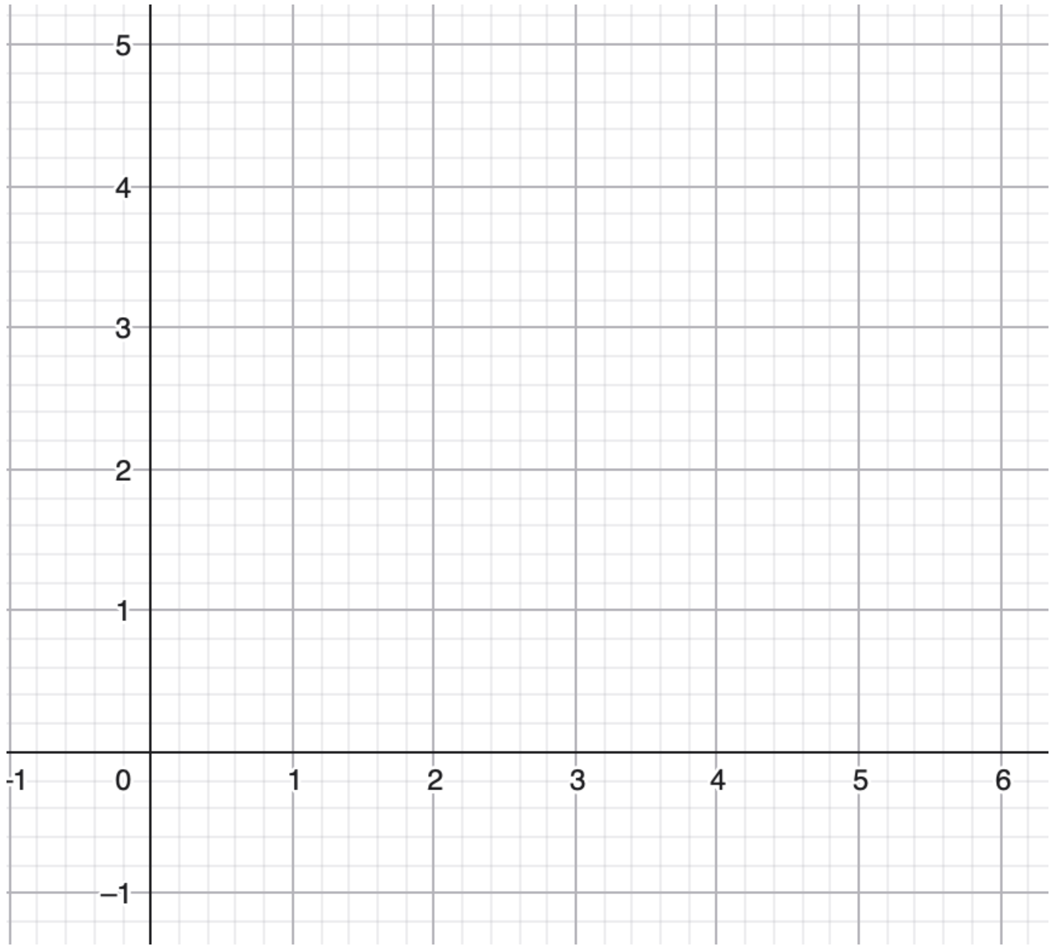
The midpoint of a line segment with endpoints at (x_1, y_1) and (x_2, y_2) is located at the point indicated by the formula below

$$midpoint = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$

Problem 5

- Write out an equation for the circle that contains a diameter with endpoints (1, 3) and (5, 1)

  - Use midpoint formula to find center
  - Use distance formula to find radius



  - Solution: $(x-3)^2 + (y-2)^2 = 5$

Combining Functions

Combining, composite, evaluating, and domains

Combining Functions – adding, subtracting, multiplying

- $(f+g)(x) = f(x) + g(x)$
  - For specific inputs add outputs
  - For general cases add functions
- $(f-g)(x) = f(x) – g(x)$
  - For specific inputs subtract outputs
  - For general cases subtract functions
- $(fxg)(x) = f(x) x g(x)$
  - For specific inputs multiply outputs
  - For general cases multiply functions

Combining Functions – division

- $(f/g)(x) = f(x)/g(x)$
  - As long as $g(x)$ does not equal 0
  - For specific inputs divide outputs
  - For general cases divide functions
  - ALWAYS check for $g(x) = 0$

Composite Functions

- $(f \circ g)(x) = f(g(x))$
  - This means we will put our inputs into our function g, then we will put the outputs of that function into our function f and get our final output

Problem 6 – Part 1

Given $f(x) = \frac{1}{x^2 - 4}$ and $g(x) = x - 5$

Find

$(f + g)(3)$

$(f \circ g)(1)$

Solutions: $-\frac{9}{5}$ $\frac{1}{12}$

Domains of Combined Functions

- Domain of (f + g)(x), (f – g)(x), (f x g)(x)
  - If and only if a real number x is in the domain of f and in the domain of g, then x is in the domain of (f+g), (f-g) and (f*g)
  - Find the domains of f and g and keep whatever they have in common
- Domain of (f / g)(x)
  - If and only if a real number x is in the domain of f and g (and g(x) is not equal to 0) then x is in the domain of (f/g)
  - Find the domains of f and g and keep whatever they have in common that does not make g(x) = 0.

Domains of Composite Functions

- Domain of (f o g)(x)
  - Consists of all real numbers x such that x is in the domain of g and g(x) is in the domain of f.
  - Find the domain and range of g
  - Find the domain of f
  - The domain of the entire composite function will be those in the domain of g whose outputs are in the domain of f

Problem 6 – Part 2

Given $f(x) = \frac{1}{x^2 - 4}$ and $g(x) = x - 5$

Find the domain of
$(f \circ g)(x)$

Solution: $(-\infty, 3) \cup (3,7) \cup (7, \infty)$

Problem 7 – Part 1

- Let the function f be defined by the set $\{(1,2),(3,5),(4,2),(5,1)\}$
- Let the function g be defined by the set $\{(2,4),(4,6),(3,3),(1,2)\}$
- Write out the ordered pairs for the function f x g.

- Solution: $\{(1,4),(3,15),(4,12)\}$

Problem 7 – Part 2

- Let the function f be defined by the set $\{(1,2),(3,5),(4,2),(5,1)\}$
- Let the function g be defined by the set $\{(2,4),(4,6),(3,3),(1,2)\}$
- Write out the ordered pairs for the function f o g.

- Solution: $\{(2,2),(3,5)\}$

Problem 8 – Part 1

- Let the function f be defined by $f(x) = \frac{\sqrt{x}}{2}$
- Let the function g be defined by $g(x) = 5x^2 - 2x + 1$
- Let the function h be defined by $h(x) = 3x^2$

- Determine the value for (f o g o h)(1) and simplify the result

- Solution: $\frac{\sqrt{40}}{2}$

Problem 8 – Part 2

- Let the function f be defined by $f(x) = \frac{\sqrt{x}}{2}$
- Let the function g be defined by $g(x) = 5x^2 - 2x + 1$
- Let the function h be defined by $h(x) = 3x^2$

- Determine the value for (g o h o f)(x) and simplify the result

- Solution: $\frac{45x^2 - 24x + 16}{16}$

Problem 8 – Part 3

- Let the function f be defined by $f(x) = \frac{\sqrt{x}}{2}$
- Let the function g be defined by $g(x) = 5x^2 - 2x + 1$
- Let the function h be defined by $h(x) = 3x^2$

- Use interval notation and write the domain of the function (g o h o f)

- Solution: $[0, \infty)$

Inverse Functions

- Inverse functions switch the inputs and outputs
- Functions f and g are inverses if and only if $f \circ g(x) = x$ and $g \circ f(x) = x$ for all x in the domain of f and g.
  - A function must be a bijection (each output has a unique input) for an inverse to exist
- We can find inverse functions by switching y and x in the equation and solving for y.
- Given a graph, you switch the x and y coordinates in order to graph the inverse

Problem 9 – Part 1

- Determine an expression for the inverse function of $f(x) = \dfrac{3x+5}{2x-2}$

- Solution: $f(x) = \dfrac{2x+5}{2x-3}$

Inverse Functions – Domains and Ranges

- The domain of the original function = the range of the inverse
- The range of the original function = the domain of the inverse
- We can find the domain of the inverse just like we would find the domain of any other function, however we then need to check if our original function imposes any restrictions

Problem 9 – Part 2

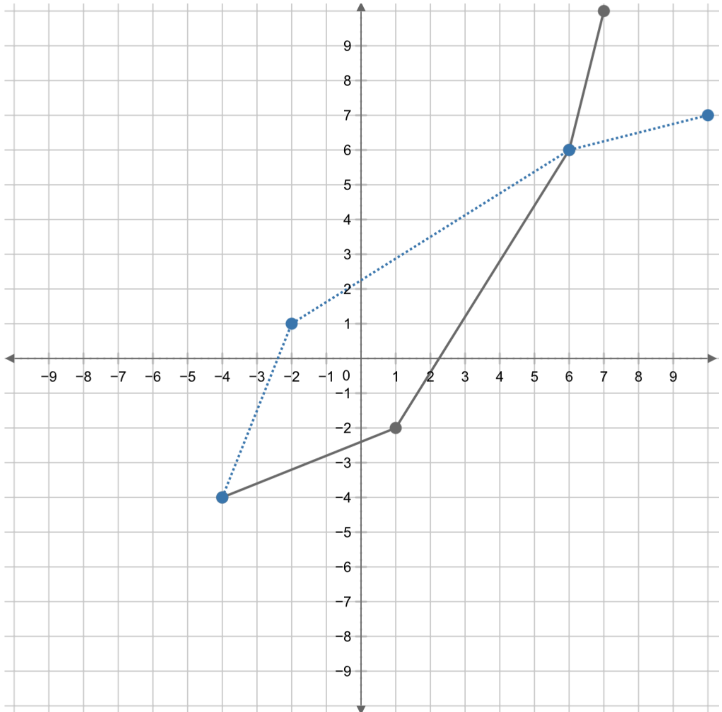
- Using interval notation write the domain and range for the inverse function of $f(x) = \dfrac{3x+5}{2x-2}$

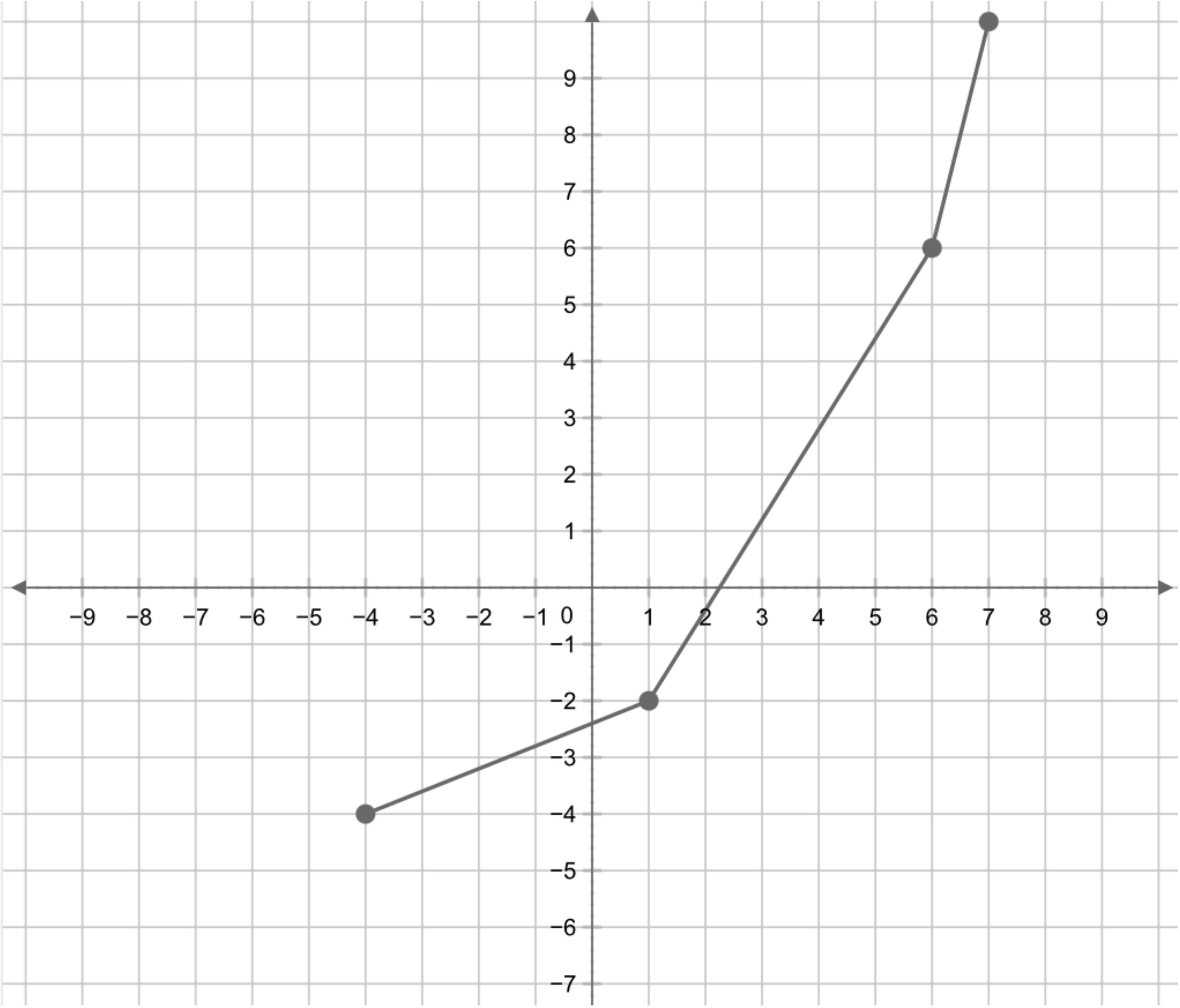
Solution:

Domain: $\left(\infty, \dfrac{3}{2}\right) \cup \left(\dfrac{3}{2}, \infty\right)$ Range: $(\infty, 1\ ) \cup (1, \infty)$

Problem 10

- The graph of function f is given
- Graph the inverse of function f





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Other Resources

- Aggie Math Learning Center
  - Visit usu.edu/math/amlc for more videos, resources, tutoring times, and recitation leader office hours