Calc 1210 CBE 3 Review

UtahStateUniversity

Aggie**MathLearning**Center

CBE 3

- Covers lessons 7-10
    - Derivatives of Trig Functions
    - The Chain Rule
    - Implicit Differentiation
    - Logarithms and Logarithmic Differentiation

Derivatives of Trig Functions

Formulas, equations, and application

Derivatives of Trig Functions

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\csc x = -cscx\ \cot x$$

$$\frac{d}{dx}\sec x = \sec x\ \tan x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

Derivatives of Inverse Trig Functions

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\csc^{-1}x = -\frac{1}{|x|\sqrt{x^2-1}}$$

Problem 1

- Find the derivative of the following function:

$$f(x) = 2\sec x + 3\sin x \tan x$$

- Solution: $f'(x) = 2\sec x \tan x + 3\cos x \tan x + 3\sin x \sec^2 x$

Problem 2

- Find f'(0) for the following function: $f(x) = \dfrac{3\tan^{-1}(x)}{\cos(x)}$

- Solution: f'(0) = 3

The Chain Rule

formula, use, and application

The Chain Rule

Taking the derivative of composite functions

$$\frac{d}{dx}[f(\boldsymbol{g(x)})] = f\,'(\boldsymbol{g(x)})\,g\,'(x)$$

Problem 3

- Find the derivative of the following function: $f(x) = \sqrt{3x+2}$

- Solution: $f'(x) = \dfrac{3}{2\sqrt{3x+2}}$

Problem 4

- Find f'(0) for $f(x) = \sqrt[5]{x^4 + 3x + e^x}$

- Solution: f'(0) = 4/5

Problem 5

- Find $f'(\sqrt{\frac{2}{3}})$ for $f(x) = 2\sin(3x^2 - 2)$

- Solution: $f'(\sqrt{\frac{2}{3}}) = 12\sqrt{\frac{2}{3}}$

Problem 6

- Find $f'(\sqrt{\frac{1}{12}})$ for $f(x) = \frac{1}{2}\cos^{-1}(3x)$

- Solution: f'(-1) = -3

Implicit Differentiation

notation, how to, and practice

Implicit Differentiation

1. Assume y is a variable representing an unknown function
2. Differentiate both sides with respect to x
3. Gather terms that contain y’ on one side and move all other to the other side
4. Factor out y’ and solve

Notes: Remember y’ = $\frac{dy}{dx}$

Also remember y is not a constant, it is a function so we must use all applicable rules of differentiation.

Problem 7

- Find $\frac{dy}{dx}$ for $\frac{y}{x} - 3x + 2y = 0$
- Differentiate both sides with respect to x
- Gather terms that contain y' onto one side
- Factor out y' and solve
- Solution: $\frac{dy}{dx} = \frac{3 + \frac{y}{x^2}}{\frac{1}{x} + 2}$

Problem 8

- Find the slope of the tangent line at (1,0) for the following equation:

$$e^y + e^x + 4xy + 7x^2 + 4y^2 = 14$$

- Use implicit differentiation to find y'
- Plug in the point
- Solution: $\dfrac{14 - e}{5}$

Logarithms and Logarithmic Differentiation

Logs, natural logs, and exponents

Derivatives of Exponential Functions

$$\frac{d}{dx}\, e^{g(x)} = e^{g(x)} \cdot g'(x)$$

$$\frac{d}{dx}\, a^{g(x)} = a^{g(x)}\, (\ln a) \cdot g'(x)$$

Derivatives of Logarithmic Functions

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\ln|x| = \frac{1}{x}$$

$$\frac{d}{dx}\log_a x = \frac{1}{(\ln a)\ x}$$

Problem 9

- Find f'(x) for $f(x) = \ln(\frac{e^{2x}}{3x+4})$

- Solution: $f'(x) = \frac{6x+5}{3x+4}$

Problem 10

- Find f'(x) for $f(x) = e^{x\sin x}\ln(3x)$

- Solution: $f'(x) = e^{x\sin x}(\sin x - x\cos x)(\ln(3x)) + \dfrac{e^{x\sin x}}{x}$

Logarithmic Differentiation

1. Natural log of both sides, replace f(x) with y
2. Use logarithmic properties to expand
3. Differentiate both sides using implicit differentiation
4. Solve for $\frac{dy}{dx}$
5. Replace y with f(x)

Problem 11

- Find f'(x) for $f(x) = (x^2 - 4x + 1)^{3x}$
- Natural log of both sides
- Expand
- Use implicit differentiation
- Solve for y'
- Replace y with f(x)
- Solution: $f'(x) = (x^2 - 4x + 1)^{3x}(3\ln(x^2 - 4x + 1) + \frac{3x(2x-4)}{x^2 - 4x + 1})$

Other resources

- Aggie Math Learning Center
  - Visit usu.edu/math/amlc for more videos, resources, tutoring times, and recitation leader office hours