

Math 1210 CBE 3 Review

UtahStateUniversity

CBE 3

- Covers lessons 7-10
 - Derivatives of Trig Functions
 - The Chain Rule
 - Implicit Differentiation
 - Logarithms and Logarithmic Differentiation

Derivatives of Trig Functions

Formulas, equations, and application

Derivatives of Trig Functions

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \cot x = -\csc^2 x$$

Derivatives of Inverse Trig Functions

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2} \quad \frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx} \cot^{-1} x = -\frac{1}{1+x^2} \quad \frac{d}{dx} \csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$$

Problem 1

- Find the derivative of the following function:

$$f(x) = 2 \sec x + 3 \sin x \tan x$$

- **Solution:** $f'(x) = 2 \sec x \tan x + 3 \cos x \tan x + 3 \sin x \sec^2 x$

Problem 2

- Find $f'(0)$ for the following function: $f(x) = \frac{3 \tan^{-1}(x)}{\cos(x)}$

- Solution: $f'(0) = 3$

The Chain Rule

formula, use, and application

The Chain Rule

Taking the derivative of composite functions

$$\frac{d}{dx}[f(\boldsymbol{g}(x))] = f'(\boldsymbol{g}(x)) g'(x)$$

Problem 3

- Find the derivative of the following function: $f(x) = \sqrt{3x + 2}$

- Solution: $f'(x) = \frac{3}{2\sqrt{3x + 2}}$

Problem 4

- Find $f'(0)$ for $f(x) = \sqrt[5]{x^4 + 3x + e^x}$

- Solution: $f'(0) = 4/5$

Problem 5

- Find $f'(\sqrt{\frac{2}{3}})$ for $f(x) = 2 \sin(3x^2 - 2)$

- Solution: $f'(\sqrt{\frac{2}{3}}) = 12\sqrt{\frac{2}{3}}$

Problem 6

- Find $f'(\sqrt{\frac{1}{12}})$ for $f(x) = \frac{1}{2} \cos^{-1}(3x)$

- Solution: $f'(-1) = -3$

Implicit Differentiation

notation, how to, and practice

Implicit Differentiation

1. Assume y is a variable representing an unknown function (so we can think about y')
2. Differentiate both sides with respect to x , using the chain rule on any terms with y in them
3. Gather terms that contain y' on one side and move all other to the other side
4. Factor out y' and solve

Notes: Remember $y' = \frac{dy}{dx}$

Also remember y is not a constant, it is a function so we must use all applicable rules of differentiation.

Problem 7

- Find $\frac{dy}{dx}$ for $\frac{y}{x} - 3x + 2y = 0$
- Differentiate both sides with respect to x
- Gather terms that contain y' onto one side
- Factor out y' and solve

- Solution: $\frac{dy}{dx} = \frac{3 + \frac{y}{x^2}}{\frac{1}{x} + 2}$

Problem 8

- Find the slope of the tangent line at (1,0) for the following equation:

$$e^y + e^x + 4xy + 7x^2 + 4y^2 = 14$$

- Use implicit differentiation to find y'

- Plug in the point

- Solution: $\frac{-14 - e}{5}$

Logarithms and Logarithmic Differentiation

Logs, natural logs, and exponents

Derivatives of Exponential Functions

$$\frac{d}{dx} e^{g(x)} = e^{g(x)} \cdot g'(x)$$

$$\frac{d}{dx} a^{g(x)} = a^{g(x)} (\ln a) \cdot g'(x)$$

Derivatives of Logarithmic Functions

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} \ln |x| = \frac{1}{x}$$

$$\frac{d}{dx} \log_a x = \frac{1}{(\ln a) x}$$

Problem 9

- Find $f'(x)$ for $f(x) = \ln\left(\frac{e^{2x}}{3x+4}\right)$

- Solution: $f'(x) = \frac{6x+5}{3x+4}$

Problem 10

- Find $f'(x)$ for $f(x) = e^{x \sin x} \ln(3x)$

- **Solution:** $f'(x) = \frac{e^{x \sin x}}{x} + e^{x \sin x}(x \cos x + \sin x)(\ln(3x))$

Logarithmic Differentiation

1. Take the natural log of both sides, replace “f(x)” with y for simplicity
2. Use logarithmic properties to expand/rearrange
3. Differentiate both sides (using implicit differentiation)
4. Solve for $\frac{dy}{dx}$
5. Replace y with the original expression for f(x) so that we have a formula just in terms of x

Logarithm Properties

- Some important logarithm properties to remember:

$$\log_b a^k = k \log_b a$$

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

Problem 11

- Find $f'(x)$ for $f(x) = (x^2 - 4x + 1)^{3x}$
- Natural log of both sides
- Expand
- Use implicit differentiation
- Solve for y'
- Replace y with $f(x)$
- **Solution:** $f'(x) = (x^2 - 4x + 1)^{3x} \left(3 \ln(x^2 - 4x + 1) + \frac{3x(2x - 4)}{x^2 - 4x + 1} \right)$

Other resources

- Aggie Math Learning Center
 - Visit usu.edu/math/amlc for more videos, resources, tutoring times, and recitation leader office hours

