Math 1210 CBE 3 Review

UtahStateUniversity

AggieMathLearningCenter

CBE 3

- Covers lessons 7-10
  - Derivatives of Trig Functions
  - The Chain Rule
  - Implicit Differentiation
  - Logarithms and Logarithmic Differentiation

Derivatives of Trig Functions

Formulas, equations, and application

Derivatives of Trig Functions

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\csc x = -cscx\ \cot x$$

$$\frac{d}{dx}\sec x = \sec x\ \tan x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

Derivatives of Inverse Trig Functions

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2} \qquad \frac{d}{dx}\csc^{-1}x = -\frac{1}{|x|\sqrt{x^2-1}}$$

Problem 1

- Find f'(0) for the following function: $f(x) = \dfrac{3\tan^{-1}(x)}{\cos(x)}$

- Solution: f'(0) = 3

The Chain Rule

formula, use, and application

The Chain Rule

Taking the derivative of composite functions

$$\frac{d}{dx}[f(\boldsymbol{g(x)})] = f\,'(\boldsymbol{g(x)})\,g\,'(x)$$

Problem 2

- Find the derivative of the following function:

$$f(x) = -5tan^{-1}(3e^{6x})$$

- Solution: $f'(x) = \frac{-90e^{6x}}{1+9e^{12x}}$

Problem 3

- Find f'(0) for $f(x) = \sqrt[5]{x^4 + 3x + e^x}$

- Solution: f'(0) = 4/5

Problem 4

- Find $f'(\sqrt{\frac{2}{3}})$ for $f(x) = 2\sin(3x^2 - 2)$

- Solution: $f'(\sqrt{\frac{2}{3}}) = 12\sqrt{\frac{2}{3}}$

Problem 5

- Find $f'(\sqrt{\frac{1}{12}})$ for $f(x) = \frac{1}{2}\cos^{-1}(3x)$

- Solution: f'(-1) = -3

Implicit Differentiation

notation, how to, and practice

Implicit Differentiation

1. Assume y is a variable representing an unknown function (so we can think about y’
2. Differentiate both sides with respect to x, using the chain rule on any terms with y in them
3. Gather terms that contain y’ on one side and move all other to the other side
4. Factor out y’ and solve

Notes: Remember y’ = $\frac{dy}{dx}$

Also remember y is not a constant, it is a function so we must use all applicable rules of differentiation.

Problem 6

- Find $\frac{dy}{dx}$ for $\frac{y}{x} - 3x + 2y = 0$
- Differentiate both sides with respect to x
- Gather terms that contain y’ onto one side
- Factor out y’ and solve
- Solution: $\frac{dy}{dx} = \frac{3 + \frac{y}{x^2}}{\frac{1}{x} + 2}$

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Problem 7

- Find the slope of the tangent line at $\left(\frac{\pi}{3}, \frac{\pi}{6}\right)$ for the following equation:

$$6\, sin\, y - 3\, cos\, x = \frac{3}{2}$$

- Use implicit differentiation to find y’

- Plug in the point

- Solution: $\frac{-1}{2}$

Logarithms and Logarithmic Differentiation

Logs, natural logs, and exponents

Derivatives of Exponential Functions

$$\frac{d}{dx}\, e^{g(x)} = e^{g(x)} \cdot g'(x)$$

$$\frac{d}{dx}\, a^{g(x)} = a^{g(x)}\,(\ln a) \cdot g'(x)$$

Derivatives of Logarithmic Functions

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\ln|x| = \frac{1}{x}$$

$$\frac{d}{dx}\log_a x = \frac{1}{(\ln a)\ x}$$

Problem 8

- Find f'(x) for $f(x) = \ln(\frac{e^{2x}}{3x+4})$

- Solution: $f'(x) = \frac{6x+5}{3x+4}$

Problem 9

- Find f'(x) for $f(x) = e^{x\sin x}\ln(3x)$

- Solution: $f'(x) = \dfrac{e^{x\sin x}}{x} + e^{x\sin x}(x\cos x + \sin x)(\ln(3x))$

Logarithmic Differentiation

1. Take the natural log of both sides, replace "f(x)" with y for simplicity
2. Use logarithmic properties to expand/rearrange
3. Differentiate both sides (using implicit differentiation)
4. Solve for $\frac{dy}{dx}$
5. Replace y with the original expression for f(x) so that we have a formula just in terms of x

Logarithm Properties

- Some important logarithm properties to remember:

$$\log_b a^k = k \log_b a$$

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

Problem 10

- Find the derivative of the following function: $f(t) = (3t + 8)^{3t}$

- Solution: $f'(t) = (3t + 8)^{3t}(3ln(3t + 8) + \frac{9t}{3t+8})$

Problem 11

- Find f'(x) for $f(x) = (x^2 - 4x + 1)^{3x}$
- Natural log of both sides
- Expand
- Use implicit differentiation
- Solve for y'
- Replace y with f(x)
- Solution: $f'(x) = (x^2 - 4x + 1)^{3x}(3\ln(x^2 - 4x + 1) + \frac{3x(2x - 4)}{x^2 - 4x + 1})$

Other resources

- Aggie Math Learning Center
  - Visit usu.edu/math/amlc for more videos, resources, tutoring times, and recitation leader office hours