Calc 1210 CBE 5 Review

UtahStateUniversity

AggieMathLearningCenter

CBE 5

- Covers lessons 14-15
  - L'Hopital's
  - Indeterminate Forms
  - Antiderivatives
  - Initial Value Problems

L’Hopital’s

Theorem, rules, and application

Review of Limits

- A limit of a function is what value the function approaches as x approaches the given value.

- Often found by plugging in the given value for x, and algebraic manipulation.

- If the right and left limits are not the same then the limit does not exist.

L'Hopital's Rule

- When taking the derivative of a quotient if the limit of *both* the denominator and the numerator is 0, the limit of *both* is infinity, or the limit of *both* is negative infinity, then the limit is equal to the limit of the derivative of the numerator divided by the derivative of the denominator

- Remember the limit of denominator must equal limit of numerator and the denominator cannot equal 0.

Problem 1

- Determine the numerical value of the limit: $\lim_{x \to 2} \frac{2x^2 + 4x - 16}{x^3 - 4x}$

- Solution: 3/2

Problem 2

- Determine the numerical value of the limit: $\lim_{x \to \frac{\pi}{2}} \frac{\cos(2x) + 1}{\cos x}$

- Solution: 0

Indeterminate Forms

Forms, rules, and how to solve

Indeterminate Forms

- If the $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = \infty$ then $\lim_{x\to a} (f(x)\, g(x))$ may or may not exist.

- If the $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$ then $\lim_{x\to a} (f(x) - g(x))$ may or may not exist.

- $\lim_{x\to a} f(x)^{g(x)}$
- For 0^0 ∞^0 1^∞

Solving Indeterminate Forms

- The first two indeterminate forms can be solved with algebraic manipulation to get your function as a quotient and then using L'Hopital's rule.

Solving Indeterminate Forms

- For indeterminate forms in the form of $\lim_{x\to a} f(x)^{g(x)}$
- Raise e to the power of the natural log of the function
- Expand using log rules
- Rewrite to be a quotient
- Use L'Hopital's Rule

Problem 3

- Determine the numerical value of the limit: $\lim_{x \to 0^+} (7x)^{4x}$

- Solution: 1

Antiderivatives

Definition, common functions, and practice

What is an antiderivative?

- An antiderivative of a function f is a function whose derivative is f.

- The collection of all antiderivatives of a function is referred to as the indefinite integral of f with respect to x.

$$\int f(x)\,dx$$

Common Antiderivatives Formulas

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C \text{ for } n \neq -1$$

$$\int k \, dx = kx + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{1}{\ln a} a^x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int k \, f(x) \, dx = k \int f(x) \, dx$$

- Don’t forget +C

Problem 4

- Evaluate the integral $$\int \frac{5}{x\sqrt{x}} + 7\sqrt[4]{x^3}dx$$

- Solution: $$\frac{-10}{\sqrt{x}} + 4x^{\frac{7}{4}} + c$$

Problem 5

- Evaluate the integral $\int (x^2 + 1)(3x + 4)dx$

- Solution: $\frac{3}{4}x^4 + \frac{4}{3}x^3 + \frac{3}{2}x^2 + 4x + c$

Problem 6

- Evaluate the integral $\int -2\sin 4x + 3e^{3x}\,dx$

- Solution: $\frac{1}{2}\cos 4x + e^{3x} + c$

Initial Value Problems

- Find the indefinite integral/s
- Plug in initial values and solve for the unknown constants.

Problem 7

- Solve the initial value problem and find f(2).

$$f''(x) = 6x + 3$$

$$f'(0) = 4, f(0) = 7$$

- Solution: f(2) = 29

Problem 8

- Solve the initial value problem and find $f(\pi)$

$$f''(x) = 3\sin x - 4\cos x$$

$$f'(0) = 2, f(0) = 2$$

- Solution: $f(\pi) = 5\pi - 6$

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Other resources

- Aggie Math Learning Center
  - Visit usu.edu/math/amlc for more videos, resources, tutoring times, and recitation leader office hours