Calc 1210 CBE 6 Review

UtahStateUniversity

AggieMathLearningCenter

CBE 6

- Covers lessons 16-18
  - Area Under a Curve
  - The Definite Integral
  - The Fundamental Theorem of Calculus

Area Under a Curve

Approximation, Riemann Sums, and Exact

Approximating Area Under a Curve

- We can approximate the area under the curve using Riemann sums. There are several different types of Riemann sums, the most common being the left and right endpoint sums.

- The formula for the right endpoint sum is: $$\sum_{i=1}^{n} f(x_i)\,\Delta x$$

- The formula for the left endpoint sum is: $$\sum_{i=0}^{n-1} f(x_i)\,\Delta x$$

Exact Area Under a Curve

- The exact area under a curve can be found by estimating our area using infinite regions
- The right-endpoint formula is: $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$

Definite Integrals

Area, Definition, Formulas, and Application

Definition of Definite Integrals

- The Right-Endpoint Definition of Definite Integral:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

- n = number of intervals

- Δx = $\dfrac{a-b}{n}$

- x_i = $a + i\Delta x$

Definite Integrals and Area

- If f is a positive function, then the definite integral is equal to the area below the curve y=f(x) and above the x-axis on the region
- If f is a negative function, then the definite integral is equal to -1 times the area above the curve y=f(x) and below the x-axis on the region
- If f is sometimes positive and sometimes negative, then the definite integral is equal to the area below the curve and above the x-axis minus the area above the curve and below the x-axis.

Properties of Integrals

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^c f(x)dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

Problem 1

- Find the right endpoint definition of the following definite integral:

$$\int_2^5 2x^3 dx$$

- Find Δx
- Find x_i
- Plug into formula

- Solution: $\lim_{n \to \infty} \sum_{i=1}^{\infty} \frac{6}{n}(2 + \frac{3i}{n})^3$

Problem 2

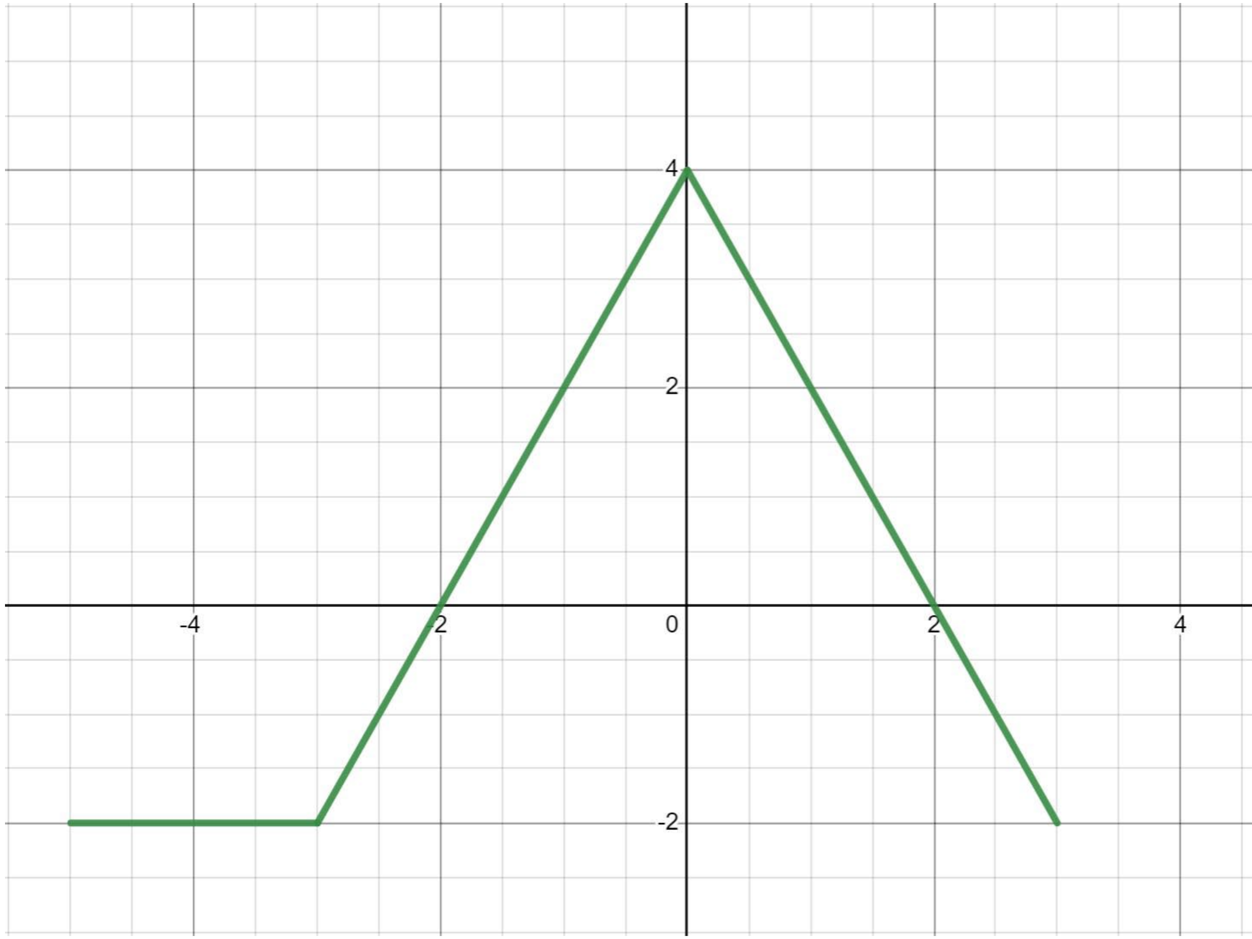
- True or False:

The integral $\int_0^4 3x^2 - 12x + 8 \, dx$ is equal to the area above the x-axis and below the curve $y = 3x^2 - 12x + 8$ on the interval [0, 4]

- Solution: False

Problem 3

- The graph of the equation y=f(x) is shown. Use the graph to evaluate $\int_{-5}^{3} f(x)\, dx$



- Solution: 2

Average Values

- If f is a continuous function on the interval [a,b], then the average value of f(x) on the interval [a,b] is:

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Problem 4

- Determine the average value of the following function over the interval $0 \leq x \leq 2$

$$f(x) = 4x^3 + 3 + e^x$$

- Solution: $\dfrac{21 + e^2}{2}$

Fundamental Theorem of Calculus

Theorem, Average Values, and Applications

Fundamental Theorem (Part 1)

- Integration as the inverse of differentiation
- Let f be a continuous function on [a,b] and let x be a point in [a,b].

$$\text{If } F(x) = \int_a^x f(t)\,dt \text{ then } F'(x) = f(x)$$

- and

$$\text{If } F(x) = \int_a^{g(x)} f(t)\,dt \text{ then } F'(x) = f(g(x))g'(x)$$

Problem 5

- If $F(x) = \int_{15}^{x^2+x} e^{t^2}\,dt$

Then what is F'(x)?

- Solution: $e^{(x^2+x)^2}(2x+1)$

Fundamental Theorem (Part 2)

- Evaluation Theorem
- Let f be a continuous function on the interval [a,b] and F be any antiderivative of f, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Problem 6

- Determine the value of the following integral $\int_{-4}^{3} 2x^3 + 4x \, dx$

- Solution: -203/2

Problem 7

- Determine the value of the following integral $$\int_0^4 \frac{3x^2}{\sqrt{x}}\, dx$$

- Solution: 192/5

Problem 8

- Determine the value of the following integral $\int_{\frac{\pi}{2}}^{\pi} \cos(2x) + 2\sin(x)\ dx$

- Solution: 2

Problem 9

- Determine the area that lies beneath the curve $y = x^2 + 1$ and over the x-axis on the interval $-1 \leq x \leq 3$

- Solution: 40/3

Other resources

- Aggie Math Learning Center
  - Visit usu.edu/math/amlc for more videos, resources, tutoring times, and recitation leader office hours