Calc 1210 CBE 7 Review

UtahStateUniversity

AggieMathLearningCenter

CBE 7

- Covers lessons 19-22
  - Substitution
  - Integration by Parts
  - Integrals Involving Exponentials and Logarithms
  - Partial Fractions
  - Length of a Curve

Substitution

U Substitution, Setup, and When to Use

When to use U Substitution

- U substitution is useful when we encounter integrals that are "more complicated"

- We know we can use u substitution if we want to find the derivative of a function that can be written as:

- $g(f(x))*f'(x)$
- Or in other words a function of a function of x multiplied by the derivative of the inside function.

U Substitution

- If we have the integral:

$$\int f(g(x))\, g'(x)\, dx$$

- We can substitute g(x) = u and then du = g'(x)dx
  - (Can solve for dx = du/g'(x))
  - Plug this into the original integral
- Then we are left with the integral of f(u) du
- Integrate and resubstitute in u = g(x)

- For a definite integral we can "transform" the bounds and not resubstitute u.

Integrals Involving Exponentials

E^x, inverse trig, logs, and applying u substitution

More Integral Formulas

$$\int e^x \, dx = e^x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C$$

$$\int a^x \, dx = \frac{1}{\ln a} a^x + C$$

$$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$\int x^{-1} \, dx = \int \frac{1}{x} \, dx = \ln|x| + C$$

- Combining these formulas with u-sub allows us to solve many different types of problems

Problem 1

- Evaluate the following integral $$\int_1^{e^2} \frac{\ln(x)^2}{2x} dx$$

- Solution: 4/3

Problem 2

- Evaluate the following integral $$\int_0^{\sqrt{\frac{\pi}{2}}} 3x\cos(x^2)dx$$

- Solution: 3/2

Problem 3

- Evaluate the following integral $\int_0^{\frac{\pi}{2}} \cos(x)\sqrt{1+\sin(x)}dx$

- Solution: $\frac{2(2)^{\frac{3}{2}} - 2}{3}$

Integration by Parts

UV Substitution, Setup, and When to Use

When to use integration by parts

- Integration by parts is like the “inverse” of the product rule.
  - (It’s kind of like the product rule for integrals)
- We use integration by parts when we want to take the integral of a product of functions that cannot be found using u-substitution or any other “Common” integral

Integration by Parts

- Formula: $$\int u\,dv = uv - \int v\,du$$
- Steps:
- Pick u and dv
  - Pick u in this order: Logs, Inverse, Algebraic, Trig, and Exponential
- Find du and v
- Plug into formula and evaluate
  - Don't forget to evaluate at endpoints

Problem 4

- Evaluate the following integral $\int_0^1 2xe^{-4x}\,dx$

- Solution: $-\frac{5}{8}e^{-4} + \frac{1}{8}$

Problem 5

- Evaluate the following integral $\int_1^e 4x^3 \ln(x)dx$

- Solution: $\frac{3e^4 + 1}{4}$

Partial Fraction Decomposition

Not on CBE but good to know and review

Setting Up Partial Fraction Decomposition

- If the degree in the numerator is greater than the denominator use polynomial division
- Completely factor the denominator
- For each linear factor we need a fraction with a constant on top
  - Ex: $\dfrac{A}{bx + c}$
- For each quadratic factor we need a fraction with a linear equation on top
  - Ex: $\dfrac{Ax + B}{cx^2 + dx + e}$
- For any squared or higher powered factors, we need a term for each power
  - Ex: $\dfrac{A}{(x+b)^3} + \dfrac{B}{(x+b)^2} + \dfrac{C}{(x+b)}$

Solving Partial Fraction Decomposition

- Multiply by least common multiple
- Group together like terms
- Set equal to original fraction
- Set up system of equations
- Solve for variables

Arc Length

Formula, and Use

Arc Length

- If f’ is a continuous function, then the length of the curve y = f(x) from x = a to x = b can be found with:

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\ dx$$

Problem 6

- Write the integral that is equal to the distance from the point $(1, 1)$ to the point $(e, 1+e^2)$ along the curve

$$y = \ln(x) + x^2$$

- Solution: $\int_1^e \sqrt{1 + (\frac{1}{x} + 2x)^2} dx$

Problem 7

- Find the exact length of the curve $y = \frac{1}{8}x + 4$
- from the point (0,4) to the point (8,5)

- Solution: $\sqrt{65}$

Other resources

- Aggie Math Learning Center
  - Visit usu.edu/math/amlc for more videos, resources, tutoring times, and recitation leader office hours