

Math 1210 CBE 1 Review

UtahStateUniversity®

CBE 1

Covers lessons 1-3

Introduction to
Limits

Limit Theorems

Continuity

Problem 1

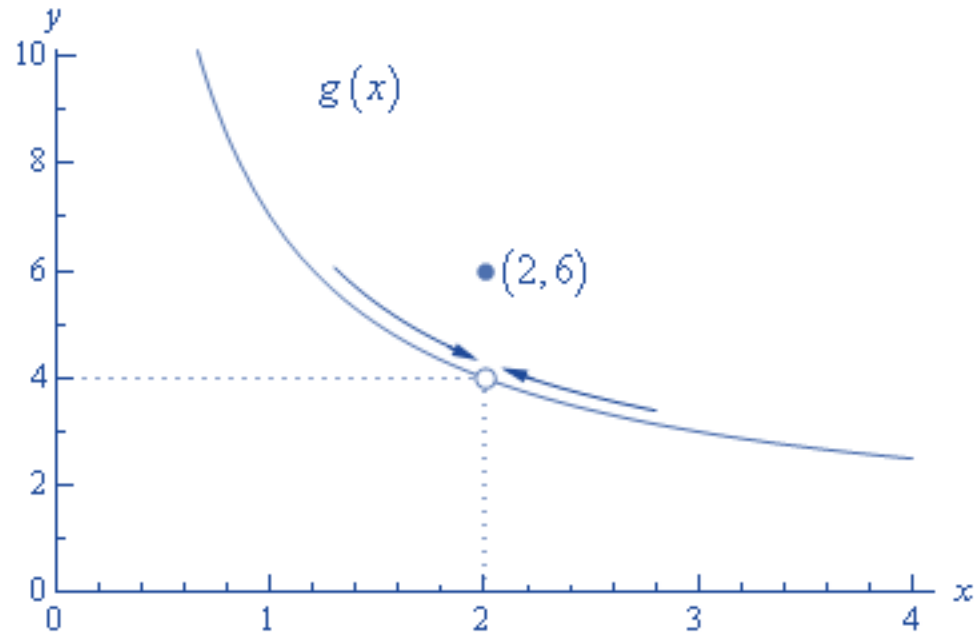
- Evaluate the following limit, $\lim_{x \rightarrow 4^+} f(x)$, for the function

$$f(x) = \begin{cases} x^2, & x < 4 \\ 3x - 1, & x \geq 4 \end{cases}$$

Solution: 11

What is a Limit?

As x approaches a value what is the function value approaching?



- This may be equal to the function value at that point, but it doesn't have to be!

Directional Limits

Left-Hand Limits	Right-Hand Limits
- Refers to x values slightly less than the given value - Notation: $-$ Example: $\lim_{x \rightarrow 3^-} 2x$	- Refers to x values slightly greater than the given value - Notation: $+$ Example: $\lim_{x \rightarrow 3^+} 2x$

Problem 2

- Evaluate the following limit

$$\lim_{x \rightarrow 3^-} \frac{2x + 5}{x^2 + 2x - 15}$$

Solution: $-\infty$

Problem 3

- Evaluate the following limit, $\lim_{x \rightarrow 2} f(x)$, for the function

$$f(x) = \begin{cases} 5x - 1, & x < 2 \\ x^3, & x \geq 2 \end{cases}$$

Solution: DNE

Introduction to Limits

A limit does not exist unless both directional limits exist and are equal

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) \Rightarrow \lim_{x \rightarrow a} f(x)$$

Problem 4

- Evaluate the following limit

$$\lim_{x \rightarrow 9} \frac{x - 9}{\sqrt{x} - 3}$$

$$\lim_{x \rightarrow 9} \frac{x - 9}{\sqrt{x} - 3}$$

Solution: 6

Problem 5

- Evaluate the following limit

$$\lim_{x \rightarrow -4} \frac{4x^2 + 8x - 32}{x^2 - 16}$$

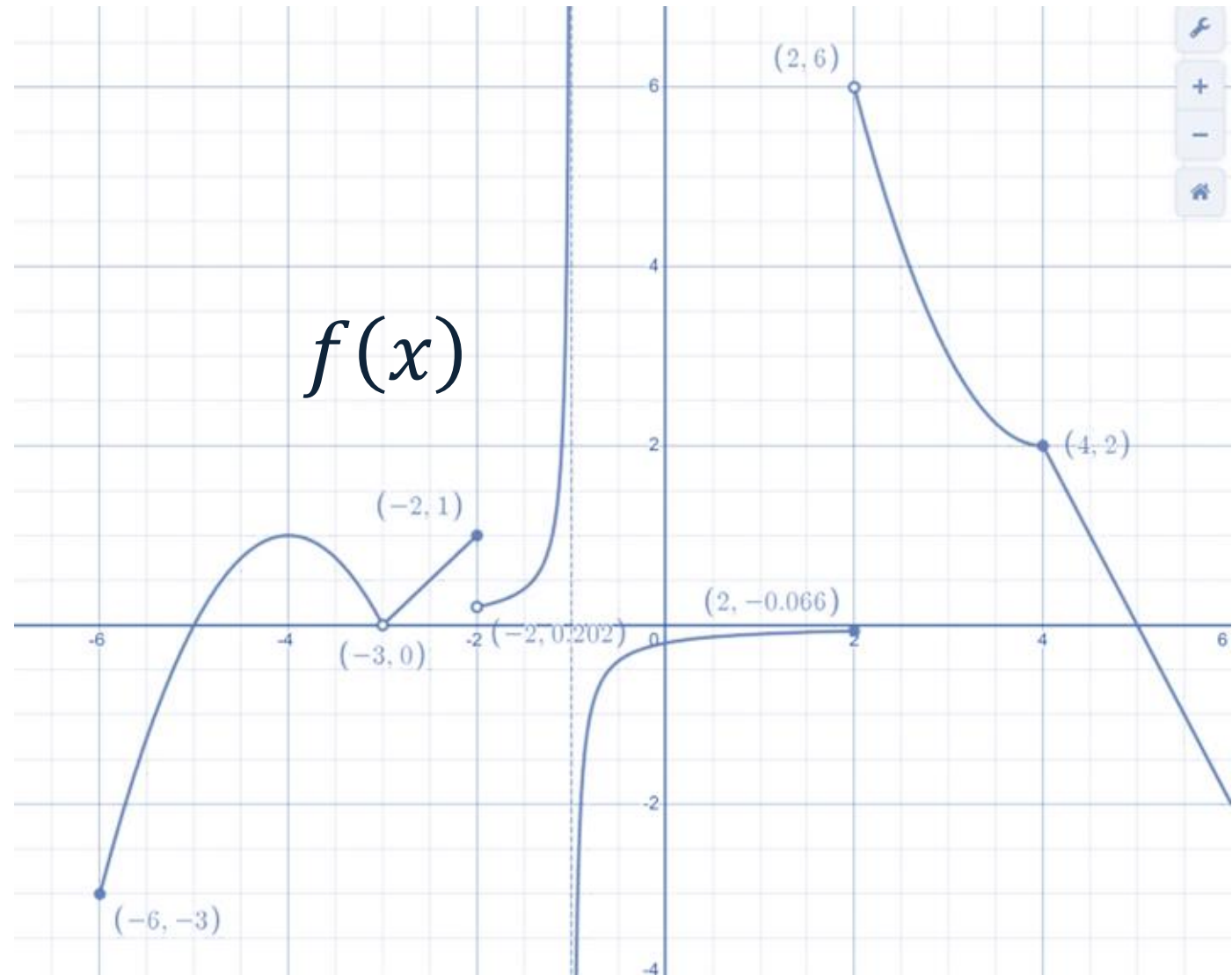
$$\lim_{x \rightarrow -4} \frac{4x^2 + 8x - 32}{x^2 - 16}$$

Solution: 3

Problem 6

- Evaluate the following limit using the graph of $f(x)$

$$\lim_{x \rightarrow 4} f(x)$$



Solution: 2

Problem 7

- Evaluate the following limit, $\lim_{x \rightarrow \frac{\pi}{2}} f(x)$, for the function

$$f(x) = \begin{cases} 3 \sin(x), & x < \frac{\pi}{2} \\ \cos(x) + 3, & x > \frac{\pi}{2} \end{cases}$$

Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} f(x)$, for $f(x) = \begin{cases} 3 \sin(x), & x < \frac{\pi}{2} \\ \cos(x) + 3, & x > \frac{\pi}{2} \end{cases}$

Solution: 3

Problem 8

- Evaluate the following limit

$$\lim_{x \rightarrow \infty} \frac{\sin(x) + 3}{5x}$$

Solution: 0

Infinite Limits

What happens to the function value as x gets really, really big?

$$\lim_{x \rightarrow \infty} f(x)$$



?

Problem 9

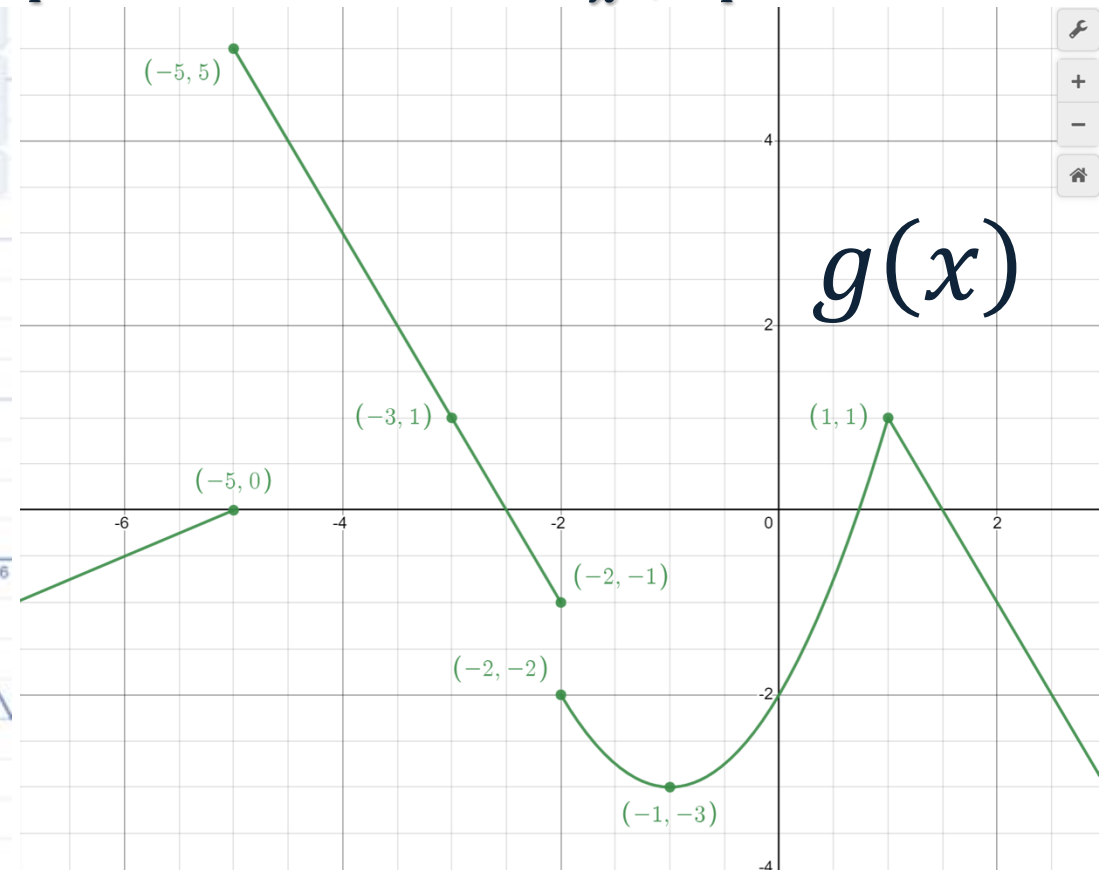
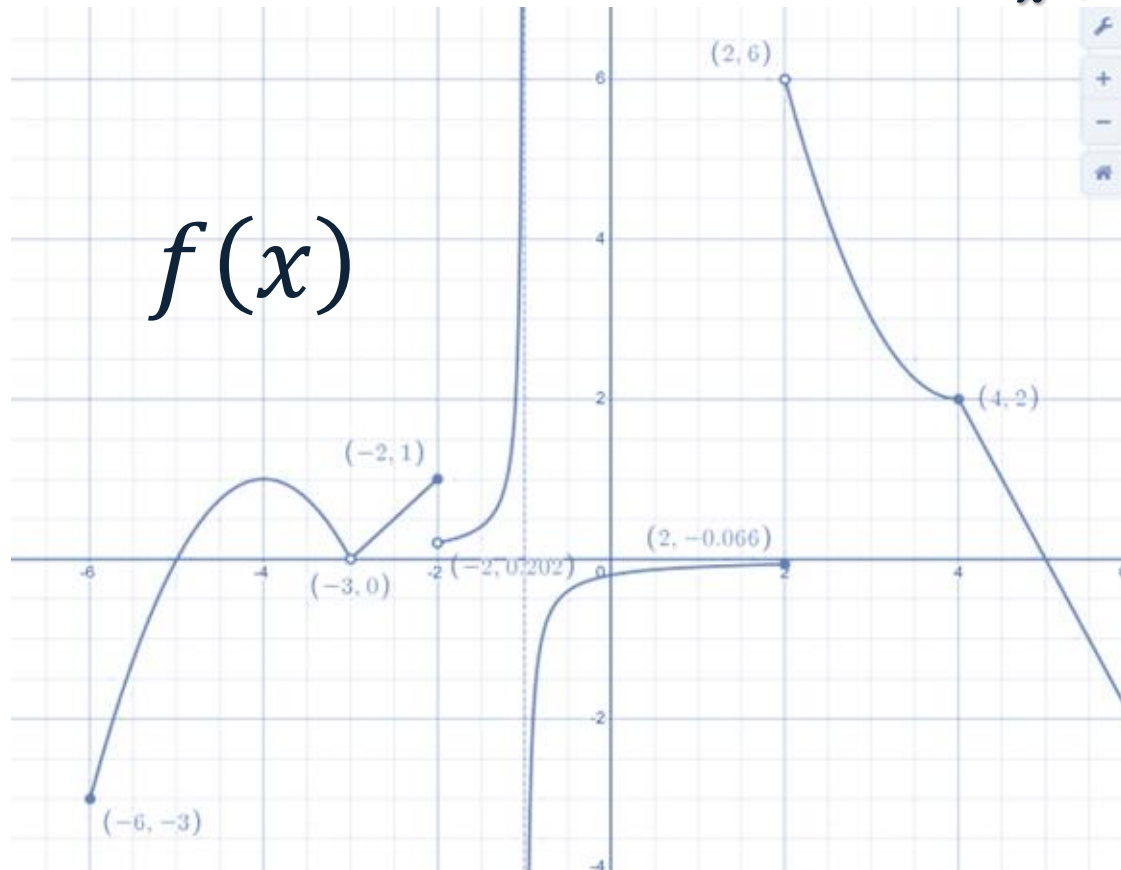
- Evaluate the following limit

$$\lim_{x \rightarrow \infty} \frac{5x^2 + 3x - 6}{x^2 + 3}$$

Solution: 5

Problem 10

- Evaluate the following limits: $\lim_{x \rightarrow -4} f(x) + g(x)$ and $\lim_{x \rightarrow -4} f(x)g(x)$



Solution: 4 and 3

Common Limit Theorems

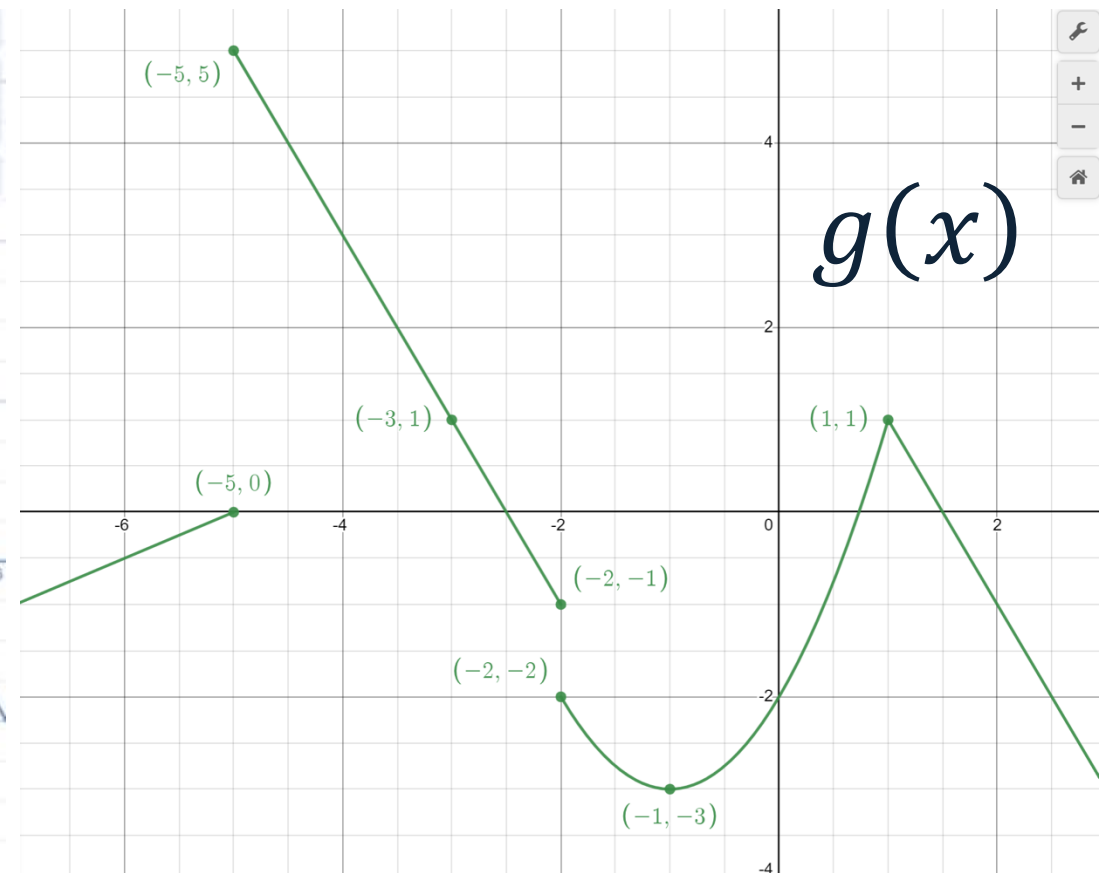
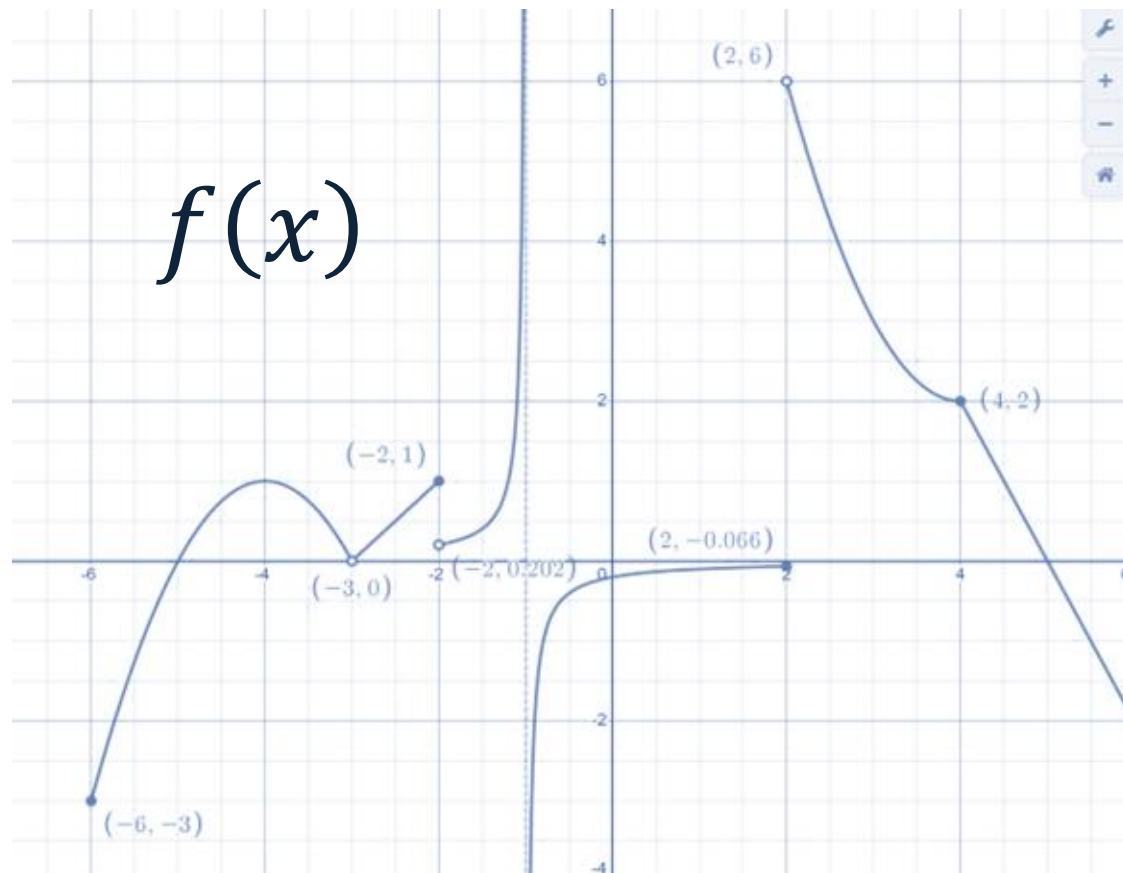
Assume that $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ both exist. Then:

$$\text{Sum Law: } \lim_{x \rightarrow a} f(x) + g(x) = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

$$\text{Product Law: } \lim_{x \rightarrow a} f(x) \cdot g(x) = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

Problem 11

- Evaluate the following limits: $\lim_{x \rightarrow -1^-} \frac{g(x)}{f(x)}$ and $\lim_{x \rightarrow -1^-} \frac{f(x)}{g(x)}$



Solution: 0 and $-\infty$

Quotient Law

Assume that $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ both exist. Then:

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}, \quad \text{as long as } \lim_{x \rightarrow a} g(x) \neq 0$$

Other Resources

Aggie Math Learning Center

- Visit usu.edu/math/amlc for more videos, resources, tutoring times, and recitation leader office hours

