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An assessment study of the wavelet-based index of magnetic storm activity (WISA) and its comparison to the Dst index

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ABSTRACT

A wavelet-based index of storm activity (WISA) has been recently developed [Jach, A., Kokoszka, P., Sojka, L., Zhu, L., 2006. Wavelet-based index of magnetic storm activity. Journal of Geophysical Research 111, A09215, doi:10.1029/2006JA011635] to complement the traditional Dst index. The new index can be computed automatically by using the wavelet-based statistical procedure without human intervention on the selection of quiet days and the removal of secular variations. In addition, the WISA is flexible on data stretch and has a higher temporal resolution (1 min), which can provide a better description of the dynamical variations of magnetic storms. In this work, we perform a systematic assessment study on the WISA index. First, we statistically compare the WISA to the Dst for various quiet and disturbed periods and analyze the differences of their spectral features. Then we quantitatively assess the flexibility of the WISA on data stretch and study the effects of varying number of stations on the index. In addition, the ability of the WISA for handling the missing data is also quantitatively assessed. The assessment results show that the hourly averaged WISA index can describe storm activities equally well as the Dst index, but its full automation, high flexibility on data stretch, easiness of using the data from varying number of stations, high temporal resolution, and high tolerance to missing data from individual station can be very valuable and essential for real-time monitoring of the dynamical variations of magnetic storm activities and space weather applications, thus significantly complementing the existing Dst index.

1. Introduction

The magnetosphere–ionosphere contains a number of current systems, such as the ring current, tail current, field-aligned current, and various electrojets in the ionosphere. These currents vary on a wide range of spatial and temporal scales and physically couple with each other. To study the complicated behaviors of these coupled current systems, the ground-based magnetometer has been a useful tool, but the recorded magnetometer data are always multi-scaled and intermittent due to the nature of these current systems.

A number of indices have been introduced to characterize the variations of specific current systems, including the Dst, AE, Kp indices, and recently a high-resolution index SYM-*H*. The Dst index was originally designed to describe the variations of the ring current that was thought to be symmetric around the earth. The idea of the Dst index was initially created by Kertz (1958, 1964) and Sugiura (1964), and a derivation scheme was proposed later (Sugiura and Hendricks, 1967). The International Association for Geomagnetism and Aeronomy

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(IAGA) version for the Dst was developed by Sugiura and coworkers (Resolution 2, p. 123, in IAGA Bulletin 27, Madrid, 1969). Although the Dst index has been used for decades in geophysics research, it never lived up to its original goal, a good description of variations of the symmetric ring current. A careful inspection of the Dst procedure would indicate that what the Dst measures are actually the storm-time enhancements of various *M–I* currents. Even though offset of the the symmetric ring current is the main component in the Dst, the contributions of the partial ring current, auroral currents, and other local-time-dependent currents can be significant (Campbell, 2004). For more details on the Dst index, readers are referred to Mayaud and others (Mayaud, 1980; Rangarajan, 1989; Kamide et al., 1998; Campbell, 1996, 1999; Rostoker, 2000; Moon et al., 2004).

One shortcoming of the Dst index is that several years of data are usually needed to produce the Dst index of good quality. To calculate the Dst index, one needs to determine the baseline for each observatory in which the secular variations and the Sq variations based on the "five quietest day" for each month are taken into account. Information about secular and Sq variations of the current year and the four preceding years is normally needed.

Another shortcoming of the Dst index is that it requires human intervention in its calculation procedure. The "five quietest days" are determined manually at each observatory, and in the case of missing data, the data from a fifth station is needed and the manual interpolation is involved. These shortcomings of the traditional Dst method can lead to difficulties in its application to real-time monitoring of storm activities and space weather.

To overcome these shortcomings in the Dst index, a wavelet-based index of storm activities (WISA) has been created by Jach et al. (2006). By applying the maximum overlap discrete wavelet transform (MODWT) method (e.g., Percival and Walden, 2000) to ground-based magnetometer data, the WISA can be automatically computed with a very flexible requirement on data stretch and it has a high tolerance for missing data. In addition, it has a much higher temporal resolution (1 m) than that of the Dst (1 h), which can better describe the dynamical variations of magnetic storm activities. The detailed description of the WISA index procedure can be found in Q1 Jach et al. (2006).

In this paper, we present a systematic assessment study of the WISA index. First, we statistically compare the WISA with the Dst for both quiet and storm periods. Second, we analyze the differences of their spectral attributes by means of the fast Fourier transform (FFT). Third, we study the variability of the WISA when it is computed with data sets of varying length and from a varying number of stations. Lastly, we assess the WISA when it is calculated with artificial missing data. Our results show that the hourly averaged WISA can describe the magnetic storm activities equally well as the Dst and, more importantly, it can complement the traditional Dst with its fully automatic procedure, flexibility with data stretch, high temporal resolution, easiness of using the data from varying number of stations, and high tolerance on missing data.

2. The WISA index

As we mentioned in the preceding section, due to the multi-scale and nonlinear nature of the *M–I* current system, the magnetometer data are non-stationary, impulsive, and their frequency spectrum changes with time. Therefore, the Fourier analysis is not well suited to the analysis of magnetometer data. Windowed Fourier analysis may offer some help, but the signals are still assumed to contain a relatively constant frequency spectrum within windows. In contrast, the unique mathematical characteristics of the wavelets make them especially suitable for decomposing the signals with time-dependent spectral features and there has been an increasing interest in using wavelets to analyze various nonlinear geophysical data (e.g., Lui and Najmi, 1997; Wei et al., 2004; Domingues et al., 2005; Haldoupis et al., 2004; Krankowski et al., 2005).

In the automatic statistical procedure of the WISA index, we use a specific wavelet technique called MODWT, which is a non-orthogonal modification of the discrete wavelet transform (DWT). The MODWT addresses as some shortcomings of the DWT, such as sample size restriction and sensitivity to the starting points of signal series. In the following, we provide a brief description of the WISA procedure.

First, the MODWT decomposes the horizontal magnetic field components into 'smoothes' and 'details' that represent the variations of different frequency levels in the recorded magnetic field. Second, the high-frequency noise, which is the small background variation (less than 0.2 nT for level 1, less than 4 nT for level 7 during the quiet period) in the high-frequency level details (levels 1–7), is eliminated by wavelet thresholding using the quantile of 0.9 and the periodic variations associated with the Sq variations are filtered from the related details. Third, the long-term trend is subtracted from the smoothes. Then, all these details and smoothes are put together and form the output for a single station. Within this output, the noise, Sq, and trend variations have been removed from the horizontal magnetic field components. Lastly, the quotients from all stations, which are obtained with dividing the variations with the cosines of their latitudes, are averaged to get the WISA index. More detailed information as well as the mathematical formula can be found in Jach et al. (2006).

3. Comparisons between the WISA and the Dst indices

In Section 2, we briefly described the statistical procedure of the WISA index. A natural question is how well the WISA represents the storm enhancements comparing to the Dst. In order to answer this question, we calculated the WISA index with magnetometer data from the four stations (shown in Table 1) used in the Dst index calculations for the period of March–April, 2001. Then we compared the WISA with the Dst in terms of their statistical properties, including difference, correlation coefficients, and root mean squared errors (RMSEs). The results are shown in Figs. 1 and 2.

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Table 1
The list of the four Dst stations

Observatory	Geographic		Geomagnetic
	Longitude(E)	Latitude	dipole latitude
Hermanus	19.22	−34.40	−33.3
Kokioka	140.18	36.23	26.0
Honolulu	201.98	21.32	21.1
San Juan to January 1965	293.88	18.38	29.9

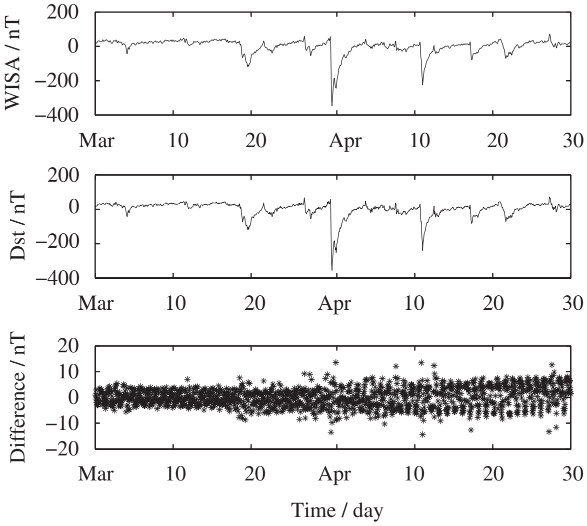

Fig. 1. The WISA, Dst indices, and their difference for the period of March–April 2001.

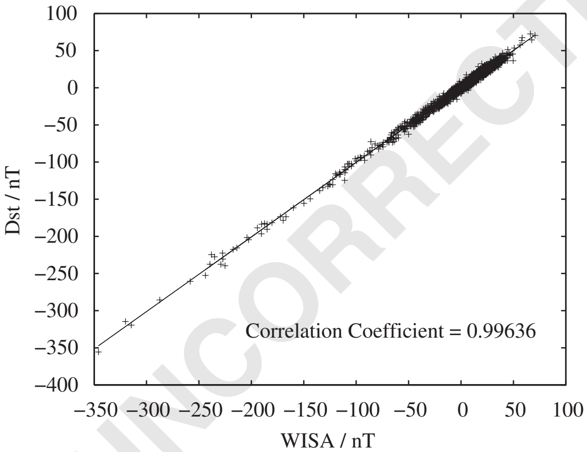

Fig. 2. The scatter plot of the WISA and Dst indices for the period of March–April 2001.

Fig. 1 shows the WISA index, the Dst index and the difference between them for the period of March and April in year 2001. Although the WISA index is calculated using the data from 2 months and the Dst index is computed with the data from more than 1 year, they are quite close to each other as shown in Fig. 1. The difference between them is around 5 nT during quiet times, and the maximum difference is less than 15 nT during storm times.

Fig. 2 shows the correlation between the WISA and the Dst indices with a high correlation coefficient of 0.996. The results indicate that the WISA and the Dst have a very good positive linear relationship.

Another statistical property we checked is the RMSE between the WISA and the Dst indices. The definition of the RMSE is as follows,

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}(\mathrm{WISA}(i) - \mathrm{Dst}(i))^2}{N-1}}$$

The RMSE is a measure of the "typical" distance between the WISA and the Dst indices. The RMSE between the WISA and Dst indices for this period is 3.820 nT. Such a small RMSE shows that the WISA index varies closely to the Dst index variations through the whole period of March–April 2001 even though there existed a very strong storm.

Furthermore, we compared the WISA and the Dst indices for quiet-time periods by using the data from July–August in year 2001. The results show the WISA and the Dst indices are still very close to each other with the difference around 10 nT except for the storm on August 18 where the difference is between 10 and 20 nT. The correlation coefficient is 0.978, and the RMSE is 2.812 nT for this period.

From the two cases described above, we can see that the WISA index is as good as the Dst index for describing the variability of the geomagnetic conditions for both storm and quiet times, but with the strength of full automation. In addition, we also compared the WISA and Dst for 1-year long data of year 2001. The results are similar to those shown in above paragraphs, which have a high correlation coefficient (0.993), low RMSE (3.951 nT), and small difference (between −20 and 20 nT). More comparisons between the WISA and the Dst indices for different periods of time are shown in Section 5 when we assess the flexibility of the WISA on data stretch.

4. FFT analysis on the WISA and Dst indices

In Section 3, we showed that the WISA can describe the geomagnetic activities of both the storm and quiet times equally well as the Dst. In order to get further details about the difference between them, we used FFT analysis to study their spectral features, with the focus on the frequency band of the Sq variations.

One of the most important steps in both the WISA and Dst procedures is removing the Solar Quiet daily (Sq) variations. We applied the FFT analysis to the WISA index, the Dst index, and the difference between them to quantitatively assess how well they remove the Sq components. Fig. 3 shows the results of these FFT analyses. We can see that the FFT results of the WISA index and the Dst index are quite similar, but there are some peaks in the FFT result of the difference. Those peaks are 24, 12, 8, and 6 h period peaks. These peaks could come from the different methods of removing the Sq variations in the two indices and the detailed explanation is as follows.

In the method of removing the Sq variations used by the Dst index, the average Sq variation for each month is first determined from the values of H component by hours for the internationally selected five quietest days of the month. Then the averages for the local hours are formed by using five local days that have the maximum overlap with the international five quietest days. And the Sq is expanded as a double Fourier series in the local time (LT) T and month number M,

$$\mathrm{Sq}(t,s) = \sum_m \sum_n A_{mn} \cos(mt + \alpha_m)\cos(ns + \beta_n)$$

The series contain 48 unknown coefficients A_{mn}, α_m, and β_n. These are determined by computing one Sq curve for each month as an average of the variation curves of the five quietest days of the month. If for a specific month, there are no ideal quiet days, the data from the same month of the preceding years are needed. Since the five quietest days are decided manually, the procedure of removing the Sq variation needs human action for the Dst index and a multi-year long data stretch in some situations.

In the method of removing the Sq variations used by the WISA index, the details referring the Sq variations are filtered to remove periodical components. This is done in 1-min resolution. Then the hourly medians are calculated for the current data stretch. There is no need to determine hourly data of the five quiet days before the subtractions of Sq variations as that in the Dst. For Fig. 3, only the data from March to April of year 2001 are used.

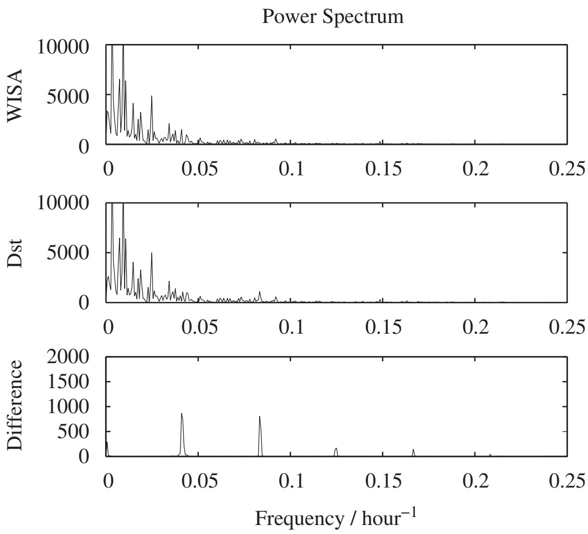

Fig. 3. The FFT results of the WISA, Dst indices, and their difference for the period of March–April 2001.

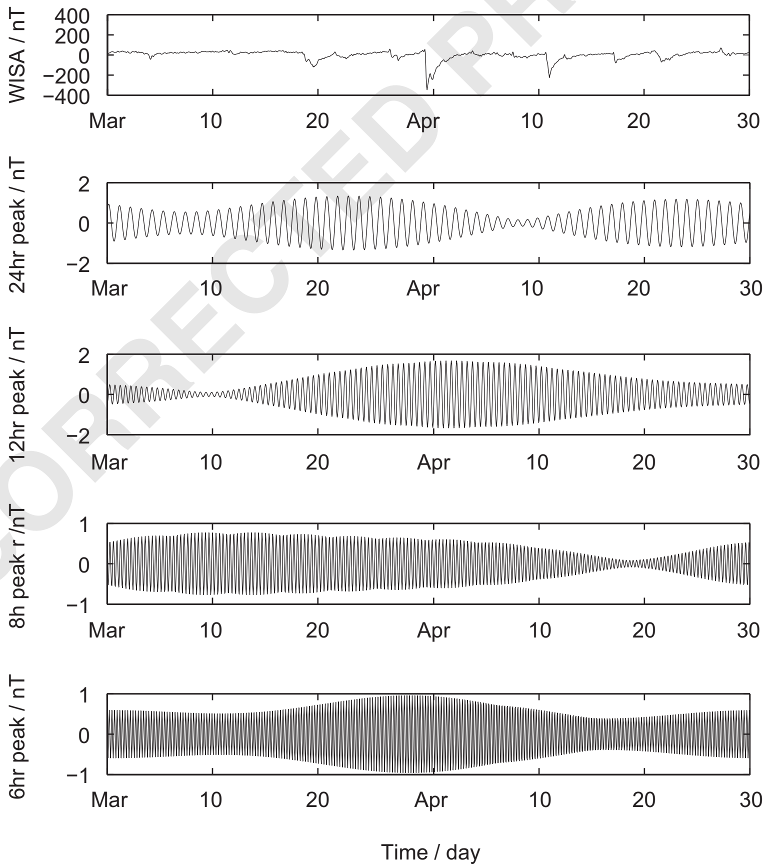

Fig. 4. The WISA index and the inverse FFT peak components of 24, 12, 8, 6 h periods for the period of March–April 2001.

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More details of the residues of Sq variations in the WISA are studied by checking the inverse fast Fourier transform (IFFT) for the periods of these peaks. We select the same wide wave band near each peak in frequency domain and transform them into time domain by IFFT. In Fig. 4, the top part is the WISA index during March–April 2001; the other four subplots are the inverse components of 24, 12, 8, and 6 h peaks during this period. All the components of IFFT of 24, 12, 8, and 6 h peaks in the WISA index are less than 2 nT, therefore the periodical residues of Sq variations are quite small and insignificant in the WISA.

In the above, both the FFT and the IFFT results show that although the WISA index removes the Sq variations automatically with flexible data stretch, the residues of the Sq variations are on the same level as that of the Dst index which uses a procedure with human actions and long data stretch. The different methods used by the WISA and Dst to remove the Sq components may cause a small difference of their Sq residues and at this point it is hard to quantitatively determine which one is cleaner with respect to removing the Sq variations.

5. Flexibility of the WISA on data stretch

After comparing the WISA with the Dst on their spectral features, we statistically assess the flexibility of the WISA on data stretch. The high flexibility on data stretch is one of the strengths of the WISA. The procedure based on the wavelet transform makes it possible to automatically remove the Sq variations from even a short data stretch, while the procedure of the Dst index requires "five quietest days" for every month which are determined manually from a long data stretch, in most cases, over 1 year. To assess this, we calculate the WISA index with data stretches of different lengths, including 1 year data, 1 month data, and even as less as 8 day data, then compare them with the Dst index. The results are as follows.

Fig. 5 shows an extreme case of these comparisons, in which the WISA index calculated with an 8-day data set is compared to the Dst index that was calculated using more than 1 year of data. In Fig. 5, the WISA index is very close to the Dst index and the difference between the WISA and Dst indices is smaller than 10 nT during quiet-time periods and is less than 20 nT during storm-time periods.

Fig. 6 shows the correlation between the WISA and Dst indices and the correlation coefficient is 0.998. They have an almost perfect positive linear relation. We also calculated the RMSE between the two indices and it is 5.255 nT, which is quite good considering the existence of a strong storm of over 200 nT during these 8 days.

Table 2 shows the results of statistical comparisons between the WISA indices calculated with different data stretches and the Dst index. The WISA and Dst indices still have highly positive linear relation for all these different data stretches. The range of difference between them is between −20 and 20 nT. The RMSE results are smaller than 5.5 nT, which means that the deviations between the WISA and Dst indices stay small.

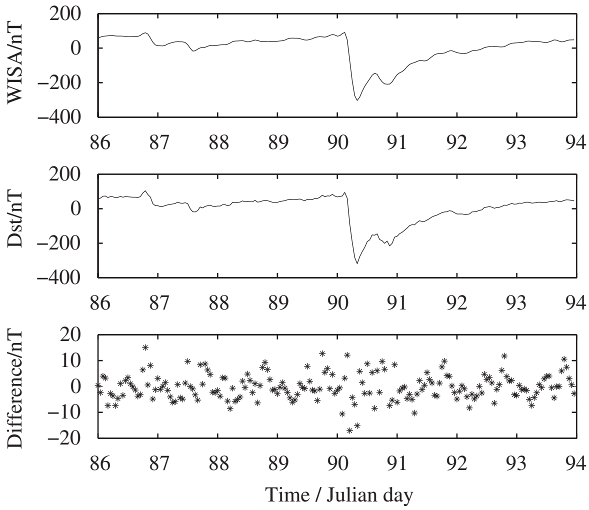

Fig. 5. The WISA, Dst indices, and their difference for the Julian days 86–93 in year 2001.

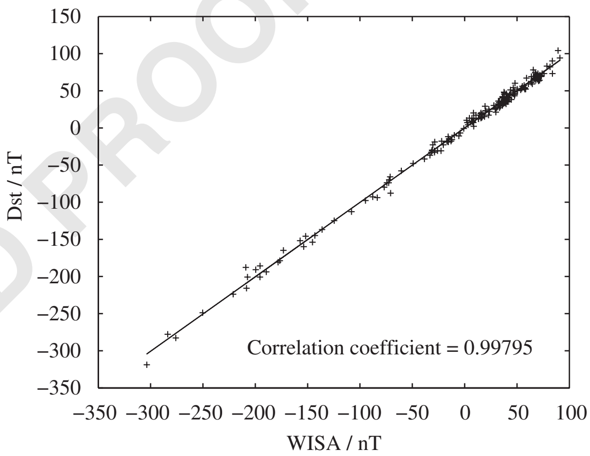

Fig. 6. The scatter plot of the WISA and Dst indices for Julian days 86–93 in year 2001.

From the above results, we can conclude that the WISA indices calculated with various data stretches work as well as the Dst for describing the enhancements of geomagnetic field *H* component during storm and quiet periods. The difference between them is always small, the RMSEs are on the order of few nTs, and the correlation coefficient is close to 1. The difference between the two indices' procedures is that the Dst index is calculated with at least 1 year data, but the WISA can be calculated by even as short as 8-day data and still have the same quality as the Dst for describing geomagnetic variations. The procedure of the WISA, which can use much less data than the Dst, makes it possible to serve as a real-time index for space weather applications.

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Table 2
Statistical comparisons between the WISA index calculated on different data stretches with the Dst index

Time periods	Range of difference (nT)	Correlation coefficient	RMSE
Year 2001	(−20,20)	0.99258	3.9505
2001.month.03–04	(−15,15)	0.99636	3.8198
2001.month.07–08	(−10,20)	0.97798	2.8123
2001.day.75–104	(−20,20)	0.99632	4.7880
2001.day.86–93	(−20,20)	0.99795	5.2546

6. Effects of varying number of stations on the WISA

In addition to the automation and the flexibility on data stretch discussed in previous sections, another assessment on the WISA is the study of effects of varying number of stations on the index calculation. This study can answer a question which is frequently asked for the Dst index—"how many stations are needed for a well behaved Dst index" (Mendes et al., 2006). Actually, the official Dst index procedure changed the required number of stations in the calculation procedure several times. The current Dst uses four stations. The hourly Dst values for the IGY (1957–1958) were based on the data from eight stations. The hourly values of the Dst for the years 1957–1970 were based on the data from three stations. Since the Dst procedure needs data of more than 1 year, it is difficult to study the effects of varying number of stations on the Dst index. For the WISA index, it is easy to study the effects of varying number of stations because its automation allows us to apply the WISA procedure to different stations easily.

First, we selected 10 stations, which consist of four original Dst stations and six low-latitude stations (listed in Table 3) and we tried to make the longitudinal distribution of the stations as uniform as possible. Then we processed the data for the period of March–April of year 2001 with the WISA procedure to calculate the *H* component enhancements and corrected them with their locations.

Second, we separated the stations into different groups and create the WISA of "varying number of stations" by averaging the *H* component enhancements in each group. For example the "2-station WISA" means the average of two stations results. The combinations of stations used for studying the effects of varying number of stations on the WISA are shown in Table 4. The stations are grouped as symmetrically as possible.

Then, the data of 10 stations are used to calculate the 10-station WISA and the 10-station WISA is compared to the Dst index for the period of March–April 2001. Fig. 7 shows the WISA, Dst indices and the difference between them. The WISA index has almost the same shape as the Dst index. The difference between them is small (around 10 nT) during the quiet-time periods, but increases to 60 nT during the storm period. The reason behind this is that the current systems contributing to the magnetic field variations around equatorial region are strongly

Table 3
The list of stations used for studying the effects of varying number of stations on the WISA

Code	Name	Colatitude	East longitude
ABG	Alibag	71.38	72.87
HER	Hermanus	124.43	19.23
HON	Honolulu	68.68	202.00
KAK	Kakioka	53.77	140.18
MBO	Mbour	75.62	343.03
MID	Midway Island	61.79	182.62
PHU	Phuthuy	68.97	105.95
SJG	San Juan	71.89	293.85
TAN	Tamanrasset	108.92	47.55
TUC	Tucson	57.82	249.27

Table 4
The combinations of stations used for studying the effects of varying number of stations on the WISA

Name	Combinations
2a-station	KAK,SJG
2b-station	HER, HON
3a-station	HON, KAK, SJG
3b-station	HER, HON, SJG
3c-station	HER, HON, KAK
3d-station	HER, KAK, SJG
4-station	HER, HON, KAK, SJG
8a-station	ABG, KAK, MBO, MID, SJG, PHU, TAN, TUC
8b-station	ABG, HER, HON, MBO, MID, PHU, TAN, TUC
10-station	ABG, HER, HON, KAK, MBO, MID, SJG, PHU, TAN, TUC

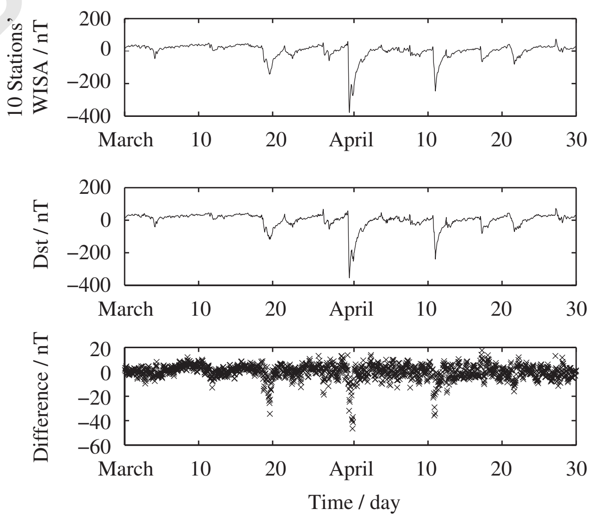

Fig. 7. The 10-station WISA, the Dst, and the difference between them for the period of March–April 2001.

asymmetric. Although in the equatorial region the ring current system is assumed to be symmetric and is a primary contributor to geomagnetic field *H* component enhancements, there are also several local-time-depen-

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dent current systems contributing asymmetrically to the geomagnetic field enhancements. When geomagnetic activities are quiet, the asymmetric enhancements are small compared to the contribution of the ring current system. However when the geomagnetic activities move to storm level, the local-time-dependent current systems increase their contributions to the enhancements of geomagnetic field *H* component. The Dst index that uses the observations of four stations is not capable of fully picking up the local enhancement component, while the 10-station WISA can.

We calculated the WISA index with 2, 3, and 8 stations data, and compare the results with 10-station WISA for three different time periods. These time periods are March–April 2001, quiet time from March 5 to 12, 2001, and storm time from March 27 to April 5, 2001. The results are shown in Table 5. According to these differences and RMSE results, we can tell that when the number of stations used in the WISA calculations increases, the results are closer to the 10-station WISA. The difference and RMSE during the quiet-time periods are smaller than those during storm-time periods. This is the evidence that the asymmetric enhancements of geomagnetic field *H* component are stronger during storm times than quiet times.

The study of the WISA with the data from varying number of stations shows that there exist asymmetric behaviors of the enhancements during geomagnetic disturbing periods. Data from four stations are not sufficient for detecting these local-time-dependent components. Data from eight stations may be sufficient to pick up the local enhancement components. The study also shows that the asymmetric behaviors of geomagnetic field are stronger during storms than quiet-time periods since the local-time-dependent components are significantly enhanced by storms. All these studies are based on the convenience of the WISA automatic procedure with flexible data stretch.

7. Effects of missing data on the WISA

With the statistical nature of the wavelet method, the WISA index can handle the data set with missing data automatically while the Dst index has to use additional observations with human intervention. In this section, the effects of missing data on the WISA are assessed by calculating the WISA index with the data sets having artificially missing data of various lengths and positions.

In order to make the periods with artificial missing data more realistic, we went through the real data of all the four Dst stations for year 2001 to find out the real distribution of missing data. In fact, Station Kakioka (KAK) and Station Hermanus (HER) have no missing data and the missing data distributions of Station San Juan (SJG) and Station Honolulu (HON) are shown in Fig. 8. According to Fig. 8, the distribution of missing data is as follows: for 1 min period, less than 50 times per year; for 10 min period, less than 24 times per year; for 30 min period, less than 10 times per year; for 1 h period, less than 10 times per year; for 3 h period, less than 10 times per year; for 12 h period, less than 10 times per year; for over 24 h period, less than 5 time per year. To realistically simulate missing data, we artificially created various periods of missing data during the months of March and April 2001 for which KAK station has no missing data. The artificial missing data periods are 10 of 1 min, 2 of 10 min, 2 of 30 min, 1 of 1 h, 1 of 3 h, 1 of 12 h, and 1 of 24 h. The resulted WISA are compared with the WISA calculated with the data without artificial missing data and the comparison results are shown in Table 6. The WISA handles the missing data with the periods shorter than 3 h quite well with its wavelet statistical procedure. The result of h missing data is still good for such an index that mainly describes the enhancements of geomagnetic field *H* component during storm time and the disturbances are normally above 50 nT. The result of 24 h missing data is noticeably different from the WISA without artificial missing data, but the chance of such a long period of missing data is only once in a year.

The above artificial missing data are mostly in the quiet-time periods. Since the WISA index is mainly used for storm activities, we also studied the effects of missing data during storm-time periods. For the storm period from March 27 to April 5 in year 2001, we applied the same types of artificial missing data periods which we used for quiet periods and repeated the same calculations as above for a missing data period of 24 h. The comparisons between the WISA with artificially missing data during storm-time periods and the WISA without artificial missing data are shown in Table 7.

Table 5
The difference and root mean squared errors (RMSEs) between 10-station WISA and other numbers of station during March– April, 2001 and the quiet time and the storm time in that period

Station combinations	March–April 2001		Quiet time (03/05–03/12/2001)		Storm time (03/27–04/05/2001)	
	Range of difference	RMSE	Range of difference	RMSE	Range of difference	RMSE
2a	(−20,60)	7.6312	(−15,10)	6.8474	(−20,60)	10.9111
2b	(−20,40)	6.3883	(−15,10)	4.5583	(−20,40)	9.7553
3a	(−30,70)	7.4707	(−15,15)	5.756	(−10,70)	10.5108
3b	(−20,40)	5.8194	(−10,15)	4.8229	(−20,40)	8.0552
3c	(−20,60)	6.5268	(−15,15)	5.0136	(−20,60)	10.4886
3d	(−20,70)	6.1322	(−15,10)	5.5164	(−20,70)	9.7718
4	(−20,50)	5.2599	(−12,8)	4.8467	(−20,40)	8.1278
8a	(−10,6)	1.5971	(−3,4)	1.1396	(−10,6)	2.4388
8b	(−14,6)	1.9078	(−3,5)	1.7118	(−14,6)	2.7278

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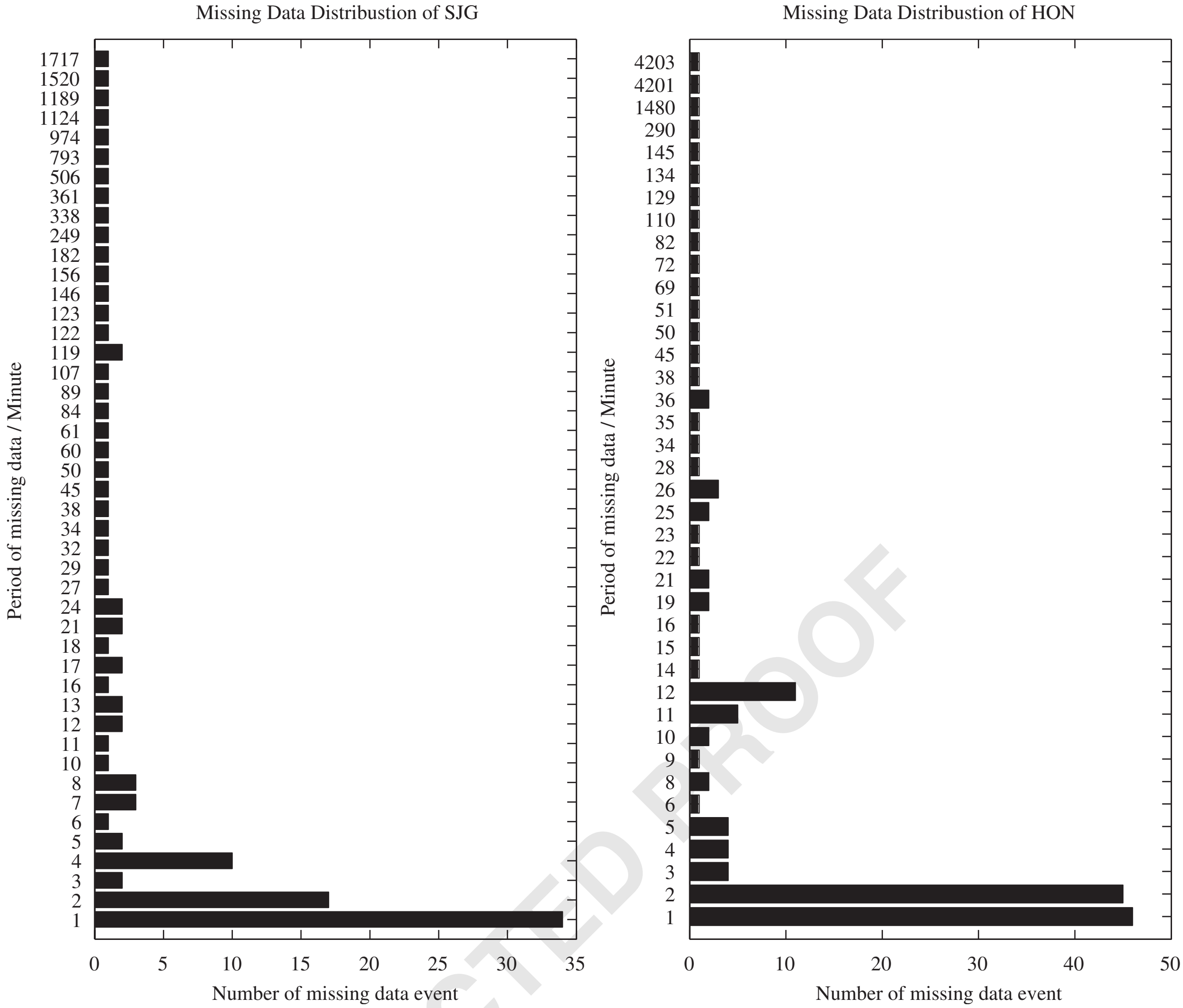

Fig. 8. Missing data distributions of SJG and HON in year 2001.

Table 6
Missing data effects during quiet time

Periods of artificial missing data	10 of 1 min	2 of 10 min	2 of 30 min	1 of 1 h	1 of 3 h	1 of 12 h	1 of 24 h
Range of difference	(−0.0025,0.0005)	(−0.007,0.001)	(−0.24,0.1)	(−0.05,0.35)	(−0.2,1.8)	(−0.5,3)	(−2,12)
Correlation coefficients	1	1	1	1	1	1	0.99992
RMSE	0.0002385	0.00030874	0.0063879	0.012001	0.067746	0.12103	0.54814

Table 7
Missing data effects during storm time

Storm	01 min	10 min	30 min	01 h	03 h	12 h
Range of difference	(−0.018,0.002)	(−0.02,0.03)	(−0.2,1.2)	(−2,0.5)	(−2,12)	(−25,10)
Correlation coefficients	1	1	1	1	0.99998	0.99955
RMSE	0.00061076	0.00095983	0.02546	0.052071	0.31832	1.4333

In Table 7, the differences and RMSEs increase by nearly one order comparing to the results for quiet-time periods. The results are still good for missing periods of less than 3 h, but for the periods over 3 h the difference are significant. In reality, the storm-time periods are much less than quiet-time periods, so the occurrence of the situation shown in Table 7 should be very rare.

In general, the WISA can automatically handle the missing data without human intervention while the Dst index needs additional data from a fifth station and manual interpolations. The WISA is still reliable for missing data less than 12 h quiet-time periods since both the average of difference amplitude and RMSE are small and the correlation show almost perfect linear relations. For storm-time periods, the WISA behave well when the periods of missing data are less than 3 h, but such a long period of missing data happens rarely during storm time. The automation of handling missing data makes is another strength of the WISA over the Dst and it is one of the crucial features for an index to be used to monitor the real-time space weather conditions.

8. Discussions and conclusions

In this work we perform a systematic quantitative assessment study on the wavelet-based index of magnetic storm activity (WISA) and statistically compare the WISA index to the Dst index with the data from various periods under various conditions. By using a wavelet-based statistical procedure, the WISA index can be calculated automatically without human intervention with very flexible data stretch. The results show that the WISA index can do equally well as the Dst index for describing the variations of geomagnetic field during both storm and quiet periods, but in addition, it has higher temporal resolution, ability of using data from varying number of stations, and high tolerance on missing data. The detailed quantitative assessment results are as the follows:

a. The comparisons between the WISA and Dst indices show that the difference between the two are consistently below 10 nT for quiet times and below 20 nT even for major storms. The statistical correlation between the two has a very good linear relationship with a correlation coefficient close to 1. The statistical deviation is very small and the values of RMSEs are between 3.8 and 3.9 nT. All these statistical results clearly indicate that the WISA describe the storm time enhancements equally well as the Dst.
b. The results of the Fourier transform analysis of the WISA and Dst indices show that the spectral features of the two indices are very similar, but there are some small peaks in the differences of the two indices in spectrum domain. These peaks may be due to the different approaches of removing the components of Sq variations in two indices. The inverse FFT results of the WISA show that the residues of the Sq variations in the index are minimal, which is around 2 nT.
c. The results from comparing the WISA calculated with varying data stretches (1 year, 2 months, 1 month, and 8 days) to the Dst show that the WISA indices are always highly correlated with the Dst index with correlation coefficients larger than 9.8 and a very small statistical deviations from the Dst. This proves that the WISA has a good flexibility on data stretch, and in contrast, the Dst may need multi-year data to produce the index of same quality.
d. The study on the effects of varying number of stations on the WISA shows that the Dst index, which traditionally uses the data from four low-latitude stations, may not be able to sufficiently pick up the local enhancement component. Eight stations can do a much better job on it. The results also show that the asymmetric enhancements of geomagnetic field *H* component can become significant during the storm-time periods.
e. The tests of computing the WISA with artificially missing data show that the WISA procedure can reasonably tolerate the missing data for less than 12 h during quiet-time periods and less than 3 h during storm-time periods.

This assessment study of the WISA index and its statistical comparisons to the Dst provides a clear quantitative picture on the quality and strengths of the WISA and its advantages over the Dst. This quantitative information would be very useful for applying the WISA method to the future studies of geomagnetic activities. With its fully automatic procedure, high flexibility on data stretch, convenience of using data from varying number of stations, high temporal resolution, and high tolerance for missing data from individual station, the WISA can be very useful and essential for real-time monitoring of the dynamical variations of magnetic storm activities and space weather applications.

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