Probability Distributions of Wavelet Coefficients of the Ground – Based Magnetometer Data for Storm and Quiet Times

Inga Maslova[1], Piotr Kokoszka[1], Lie Zhu[2], Jan J. Sojka[2]

[1]Mathematics and Statistics Department, [2]Center for Atmospheric and Space Sciences, Utah State University, Logan

Abstract

The ground-based magnetometer network has long been a powerful tool for monitoring and observing the variations of the currents flowing in the magnetosphere-ionosphere (M-I) system. These current variations directly reflect the response of the M-I system to the solar wind driver and are closely connected to various nonlinear dynamic processes in the M-I system, including storms and substorms. Due to the multiscale and nonlinear natures of the M-I current system, the time series of magnetometer data are non-stationary and their frequency behavior changes over time. They are therefore not amenable to traditional time domain or spectral (Fourier) analysis. In recent years, various new mathematical techniques have been developed to analyze magnetometer data and the wavelet technique has stood out as being particularly relevant.

In order to correctly make statistical inference based on wavelet analysis, the wavelet coefficient distributions of magnetometer data must be examined. In this work, we applied the discrete wavelet transform to the 1 min magnetometer data from March 2001 to April 2001, and then used various statistical techniques to analyze the probability distributions of the wavelet coefficients and estimate their tail indexes. It is found that the distributions of the wavelet coefficients of the magnetometer data for both storm and quiet times is highly non-normal and can be classified as being heavy tailed and the tail index values are centered around 2. This means that the probability of exceptionally large wavelet coefficients is much higher than implied by the standard statistical theory based on the normal distribution. It is also found that the tail indexes for storm times are on average smaller than those of quiet times, which reflects the stronger impulsive and non-stationary features in magnetometer data during storm times.

Background

- Magnetometers are widely used to monitor variations of current system in space.
- Magnetometer records are non-stationary.
- Traditional **Fourier** analysis — **not applicable!**
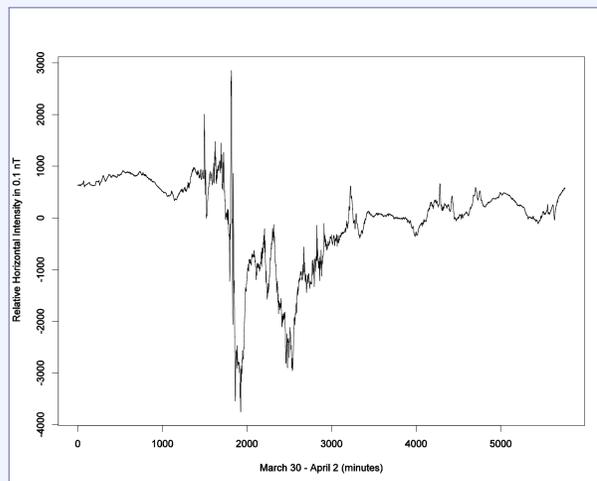- **Wavelet analysis has potential applicability!**

Figure 1. INTERMAGNET data used in current analysis: Relative Horizontal intensity in 0.1 nT, Boulder station March 30 - April 2, 2001

Data

In this work ten 3-day time series from March to April, 2001 were used. Five of them correspond to quiet time and five – to stormy time (see Figure 5).

To illustrate the procedure a time series of length n = 4320 from March 30 to April 2, 2001 was chosen (Figure 1).

Objectives

1. Examine distributions of the wavelet coefficients.

   Address the hypothesis that they are not Gaussian, rather heavy-tailed.

   Most classical methods will fail!

2. Examine the difference in distribution during stormy and quiet times.

3. Identify whether wavelet coefficients are heavy-tailed. If that is the case estimate the tail index. Tune tail index estimation methods for magnetometer data (Figure 4).

Discrete Wavelet Transform (DWT)

Suppose $X = [X_0, X_1, ..., X_{N-1}]^T$ is an observed time series.

DWT is a column vector W of the same length as observations:

$$W = [d_1^T, d_2^T, ..., d_J^T, s_J^T]^T.$$

W is obtained from vector X by multiplying it by an orthogonal matrix.

Wavelet coefficients $d_{j,k}$ reflect the oscillations of the observations around time $2^j k$ and at scale 2^j (see Figure 3).

Transformation objective:

View magnetometer data at different scales.

Heavy tailed distributions

The probability tails of a heavy-tailed random variable approach zero much slower than in the case of the normal distribution.

There is a large probability that an observation is far away from the mean, as in Figure 2 (b).

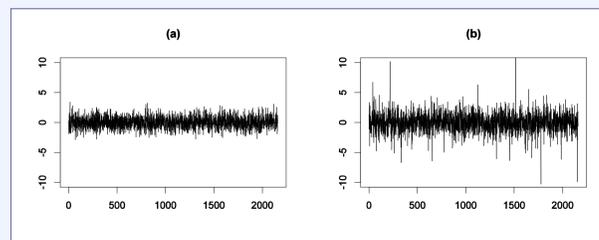If tail index is less than 2 – infinite variance distribution.

Figure 2. (a) Gaussian random variables; (b) Heavy tailed random variables with tail index 1.85

Methods

1. Visual detection of heavy-tailed data:
   - Converging variance test (Figure 4 (b))
   - QQ-plots (Figure 4 (c))
2. Tail index estimation:
   - Hill estimator (Figure 4 (d))

Results

- Wavelet coefficients come from a heavy tailed distribution (see Figure 4).
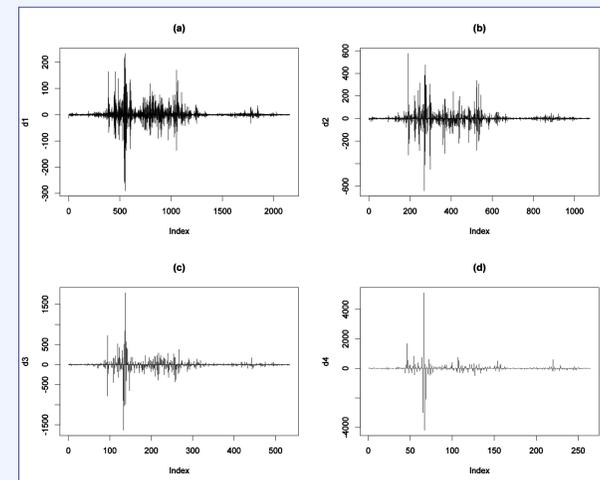- Tail index of the DWT coefficients in Figure 3 is in the interval from 0.5 to 2.5.

Figure 3. DWT coefficients of magnetometer records given in Figure 1 at four different levels: (a) 2 min; (b) 4 min; (c) 8 min; (d) 16 min.
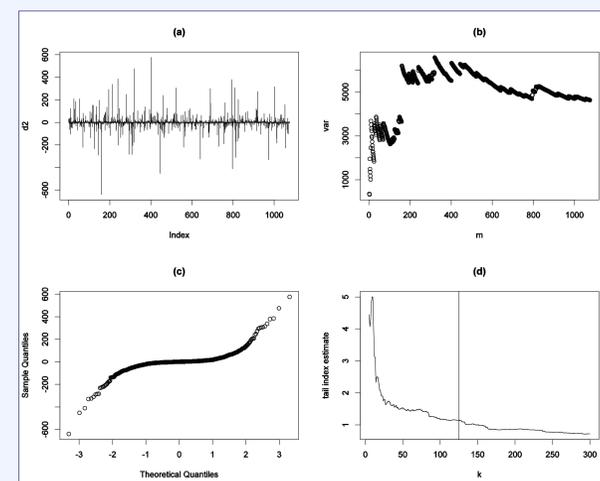

Figure 4. (a) Randomized DWT coefficients d_2; (b) Variance plot of randomized d_2; (c) QQ-plot of randomized d_2; (d) Hill plot of randomized d_2

Conclusions

- Wavelet coefficients of H – component are heavy tailed (holds for storm and quiet times, other latitude stations).
- Tail index is in the range from 1 to 3.
- Average tail index of stormy periods is slightly smaller than the corresponding average of quiet periods (see Figure 5).
- Difference between average tail index estimates for levels 2 and 3 (4 – 8 minutes) of stormy and quiet times is statistically significant.
- Applying statistical techniques make sure they are robust to large departures from Gaussianity and take into account the presence of heavy probability tails.
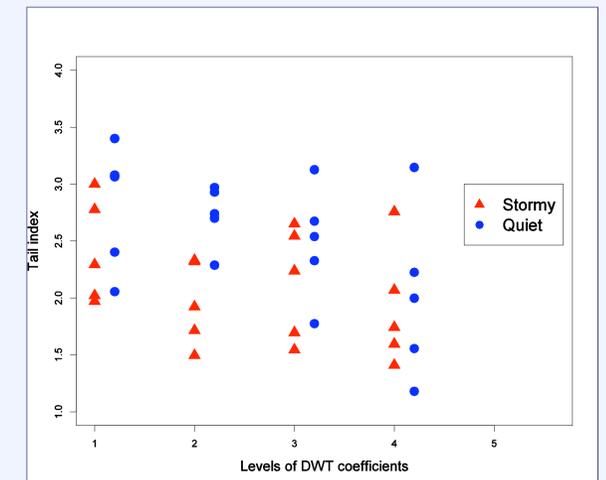

Figure 5. Hill estimates of the wavelet coefficients of magnetograms during the stormy and calm periods in March – April, 2001.

Future work

- Apply the same analysis to a large database, in order to statistically confirm the systematic decrease of the tail index value from quiet to storm periods.
- Explore tail index behavior for wavelet coefficients at scale j = 2, 3.

References

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Percival, D. B., and A. T. Walden (2000), *Wavelet Methods for Time Series Analysis*, Cambridge University Press, Cambridge.

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Contact information

For further information please contact Inga Maslova (Inga@cc.usu.edu). Manuscript on which the poster is based is available upon request. Poster is available at http://cc.usu.edu/~inga/research.htm.