

Improved Functional Wavelet-Based Index of Magnetic Storm Activity

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Motivation

- Jach et al. (2006) introduced an automated, easy to use methodology of wavelet-based index of storm activity (WISA) construction.
- Lika in Dst in WISA daily variations are assumed to be constant (over one month).
- Xu and Kamide (2004) used the method of orthogonal components to analyze the daily magnetic variations.
- Chen et al. (2007) used principal component techniques to separate pure Sq.

Objectives

- Improve the existing WISA index.
- Design a procedure to remove a *non-constant* daily variation.

Our methodology is novel in several ways:

- We isolate the daily periodic component using wavelet decomposition (MRA levels 8, 9, 10).
- Unlike Xu and Kamide (2004) and Chen et al. (2007), we use several stations to deconvolute the storm activity from daily variation.
- Construct a statistical procedure that separates storm activity from daily periodic component.
- Use functional canonical correlations to compare the improved index to WISA.

Wavelet decomposition

Let, $X_s = \{X_{s,t}, t = 1, \dots, N\}$ be the magnetogram recorded at station s , N is the sample size in minutes.

We can write it as

$$X_s(t) = \sum_{j=1}^J D_{s,j}(t) + S_{s,j}(t),$$

where $D_{s,j}$ – details, $S_{s,j}$ – smooth.

Here, $j = 1, \dots, J$ is the multiresolution analysis (MRA) level. The details capture the part of the records that correspond to the frequencies in the range from 2^{-j-1} to 2^{-j} cycles per minute.

For further details see Chapter 5 of Percival and Walden (2000)

Removal of the daily periodic variation

Step 1. MRA decomposition

Extract MRA details from the raw magnetometer records at levels $j = 8, 9, 10$, using MODWT and LA(8) filter.

$$D_{s,P}(t) = D_{s,8}(t) + D_{s,9}(t) + D_{s,10}(t)$$

Level $j = 8$ captures 6 hour periodic component, $j = 9 - 12$ hour component, and $j = 10 - 24$ hour component.

From now on, work with the details $D_{s,P}$.

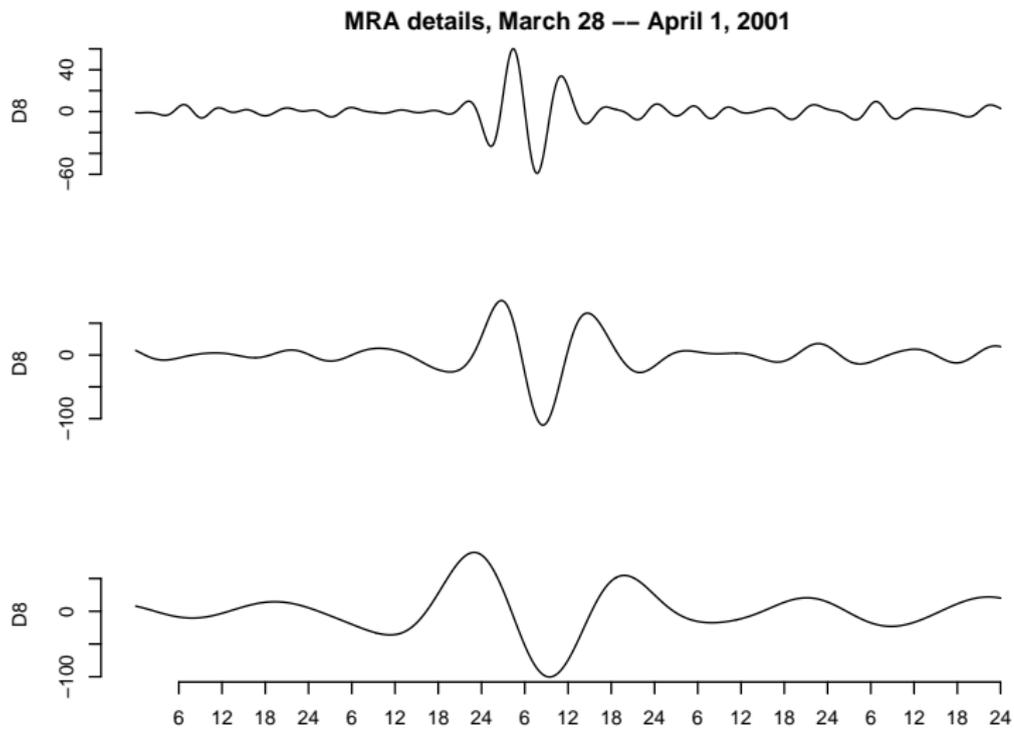


Figure: MRA details D_8 , D_9 , D_{10} , Mar 28 - Apr 1, 2001, HON.

Isolating storm signature

Step 2. Remove the average of all stations

Find the average of all stations used in the study and remove it from all data.

$$\bar{D}_P(t) = \frac{1}{m} \sum_{s=1}^m D_{s,P}(t),$$

This average will be a part of the index.

MRA, D8+D9+D10, March 28 – April 1, 2001

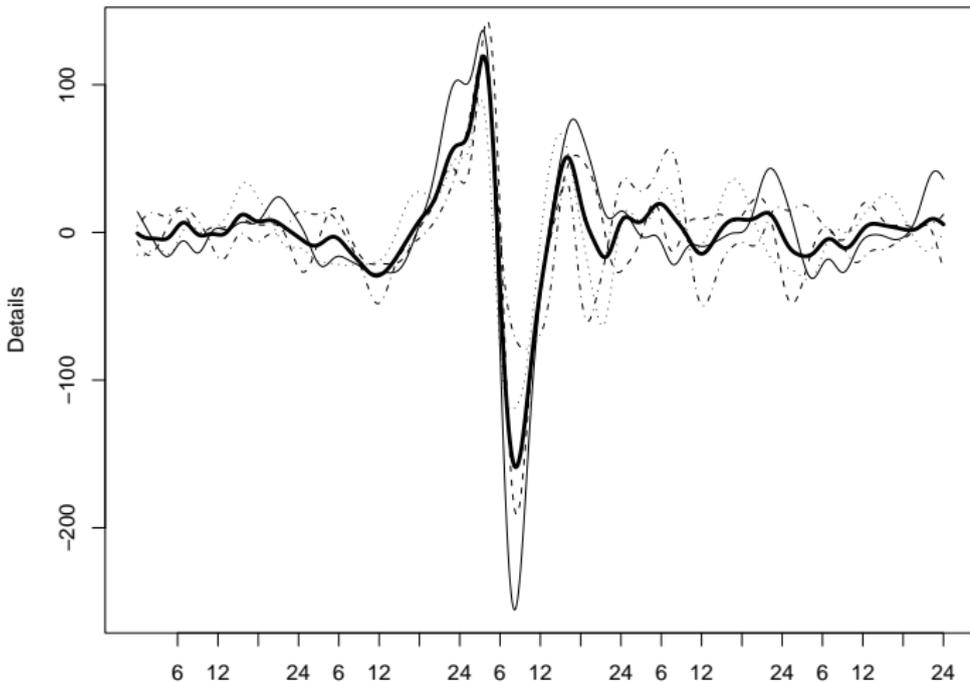


Figure: $D_{s,P}$ components and their average (thick line) of 4 Dst stations: HON, KAK, SJG, HER, March 28 - April 1, UT

Extracting the residual storm activity

Step 3. Convert data to functional object

Split the data into daily observations (functions) in UT.
The observations are daily functions (not points).

Step 4.

Compute the daily first principal component, $u_{s,1}(t)$, (harmonic) for each station s separately (use $D_{s,p}^c(t)$ – data after average removal).

Extracting the residual storm activity

Step 5. Daily periodic component.

Denote,

$$P_s(t) = \mu_s(t) + \gamma_{s,1}^* u_{s,1}(t), \quad t = 1, \dots, 1440,$$

where $\mu_s(t)$ is the daily mean, $\gamma_{s,1}^*$ are the *filtered* scores, and $u_{s,1}(t)$ is the first harmonic for station s .

Replace the largest 5% of scores by the median score only if the extreme value is captured by all stations.

Improved index

Step 6. Storm activity for station s .

The contribution of station s to the storm signature is

$$I_s(t) = \bar{D}_P(t) + (D_{s,P}^c(t) - P_s(t)),$$

where $t = 1, \dots, N$.

$\bar{D}_P(t)$ – average of MRA details of all stations used in the study.

$D_{s,P}^c(t)$ – data after we removed the average $\bar{D}_P(t)$

$P_s(t)$ – the extracted daily periodic component.

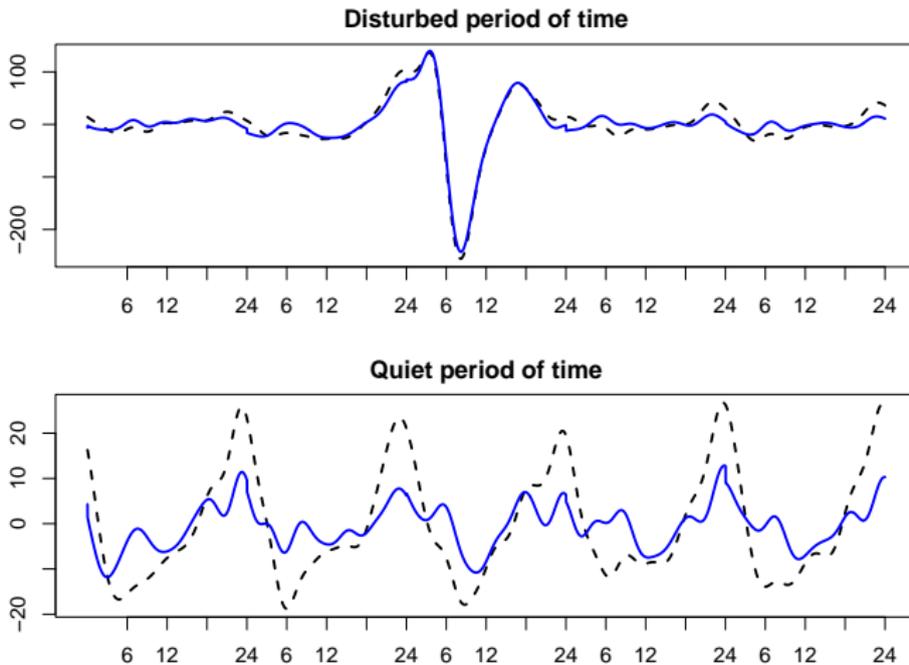


Figure: Improved pre-index (solid blue line) and original data (dashed black line) for one station

Functional canonical correlations

Let, $\rho(s_1, s_2)$ be the functional canonical correlation between $I_{s_1}(t)$ and $I_{s_2}(t)$.

The higher the value of $\rho(s_1, s_2)$ the closer the pre-indexes of two stations, i.e. more local time features were removed and global storm signature isolated.

Compare to WISA

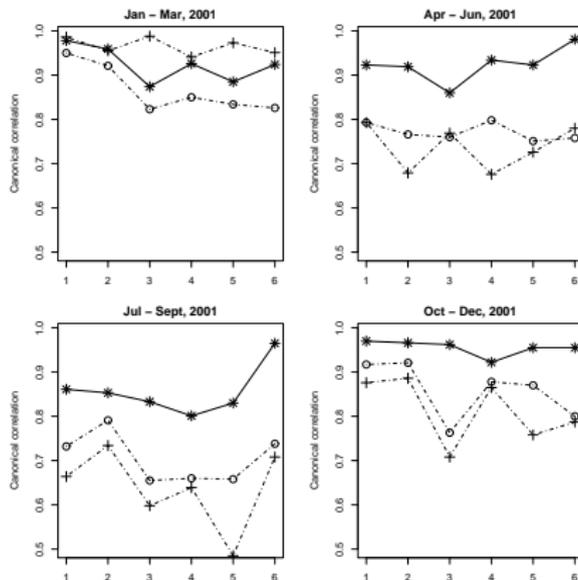


Figure: Canonical correlations for the new method (star), new method without centering (cross) and WISA (circle), applied to all combinations of four Dst stations.

Conclusions

- Our method removes a *non-constant* daily periodic variation.
- Clear improvement from existing WISA and Dst: global storm activity is isolated in a more efficient and cleaner way.

References

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Thank you!